

DATA-DRIVEN COMPUTATIONAL INVESTIGATION OF FLUID-FLOW AND HEAT-TRANSFER CHARACTERISTICS IN AN IN-LINE TUBE BUNDLE UNDER PULSATING CROSS-FLOW

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ABSTRACT

Reciprocating pulsating flow offers a controllable approach for improving convective heat transfer in tube-bundle systems. This study combined secondary experimental data with computational modelling to evaluate 180 regimes covering 18 frequency-amplitude-duty-cycle combinations across ten Reynolds-Prandtl operating blocks. Factorial ANCOVA, HC3 robust inference, uncertainty-weighted regression, leave-one-block-out validation, dimensionless correlation development, bootstrap analysis, and multi-response optimization were applied. Pulsating flow improved heat transfer in 95.56% of the regimes, while 81.67% achieved at least 10% enhancement. Mean Nusselt number and heat-transfer enhancement ratio were 58.87 and 1.247, respectively. Frequency and amplitude ratio were the strongest controllable factors, while a duty cycle of 0.2 performed better than 0.5. Blocked validation yielded R^2 values of 0.9779 for Nusselt number and 0.7782 for enhancement, with corresponding mean absolute percentage errors of 2.56% and 4.76%. Strouhal-based correlations achieved blocked R^2 values of 0.9593 and 0.8431. The strongest tested condition, $f = 0.450\text{Hz}$, $A/D = 15$, and $\psi = 0.2$, produced $Nu = 71.63$ and $Nu_p/Nu_{st} = 1.520$, approximately 52.7% above the low-pulsation reference. Its selection across all validation folds and 189 sensitivity scenarios confirmed robust thermal optimality within the measured domain.

KEYWORDS: Nusselt number; heat-transfer enhancement; dimensionless correlation; uncertainty-aware modelling; operating-regime optimization

1. INTRODUCTION

Efficient control of fluid flow and convective heat transfer remains a central requirement in thermal and mechanical engineering. Tube bundles are widely used in heat exchangers, condensers, boilers, evaporators, process equipment, and thermal-management systems because they provide a large heat-transfer area within a compact volume. Performance of these systems depends on the interaction among flow velocity, fluid properties, tube arrangement, boundary-layer development, wake formation, and thermal diffusion. Conventional steady-flow operation is relatively easy to design and control, but it may provide limited fluid mixing in low and transitional Reynolds-number regimes. Active flow modulation offers a possible means of improving thermal performance without changing the primary geometry of the system.

Pulsating flow is an active heat-transfer enhancement technique in which fluid velocity or flow rate varies periodically with time. Periodic acceleration and deceleration can interrupt boundary-layer development, alter flow separation, intensify wake motion, and increase exchange between the near-wall fluid and the main flow. Early investigations of pulsating pipe flow showed that its thermal behavior cannot be explained by Reynolds number alone because the response also depends on pulsation frequency, amplitude, waveform, and thermal development length [1]. Experimental results also showed that the amplitude of pulsation affects convective heat transfer, but the enhancement is not always monotonic with all the operating conditions [2].

In the study of pulsating flows, the timescales that are important for momentum and heat transport are several and interact, making dimensionless analysis very significant. The Reynolds number, Re , is a ratio of inertial forces to viscous forces and the Prandtl number, Pr , is a ratio of momentum diffusivity to thermal diffusivity. The Strouhal number, St , is a combination of the frequency of the pulsation and the convective timescale. Thermal diffusion in a characteristic time is indicated by Fourier number, Fo . Amplitude ratio (A/D) is the measure of imposed displacement in comparison to the tube diameter and duty cycle (ψ) is the measure of the asymmetry of the wave form. Analytical study of laminar pulsating flow has revealed that time-averaged heat transfer is a function of coupling between the pulsation and the thermal-development conditions and

not just a function of pulsation frequency [3]. Other tube-flow studies have also shown that the effect of pulsation can be different depending on the boundary conditions applied and the ratio of the flow momentum and energy transport timescales [4].

The additional complexity that is added to the flow around a cylinder due to external cross-flow is also due to the interaction between periodic forcing, flow separation, and vortex shedding. When the imposed oscillation is of the same frequency as the natural wake instability, a lock-on condition can occur, where the imposed oscillation is dynamically coupled with the shedding of the vortices. Experimental studies in the vicinity of a heated cylinder have correlated certain pulsating Strouhal numbers to the development of organized vortical structures and increased heat transfer coefficient from the surface [5]. This kind of results shows that there might be an optimum pulsation frequency in a certain flow domain. The frequencies lower than this range may not be sufficient to provide adequate boundary-layer disruption, and frequencies higher than this range may not allow formation of the large-scale vortices.

Repeated wakes, restricted passages, and interactions between successive rows of tubes in a tube bundle make it difficult to obtain physical insight in the heat-transfer behavior around an isolated cylinder. The velocity and turbulence of downstream cylinders are influenced by upstream cylinders. Bundle performance is determined by local gap acceleration, wake impingement, recirculation and thermal-boundary-layer redevelopment. The experimental investigation of a heated cylinder in pulsating crossflow has demonstrated the changes in the heat transfer distribution around the cylinder circumference in the separated-flow region [6]. Direct investigation of pulsating flow through a tube bundle has also shown that the heat transfer within the bundle is also very sensitive to the pulsation parameters and the pulsation flow development across the successive rows of the tube bundle [7].

Pulsation effects are also sensitive to the imposed waveform. Sinusoidal, square-wave, intermittent, and asymmetric waveforms produce different acceleration histories even when their nominal frequency is similar. Numerical analysis of pulsating flow around a heated square cylinder has shown that changes in frequency and amplitude alter vortex formation, wake length, and time-averaged heat transfer [8]. Experimental results for turbulent pulsating cross-flow around a circular cylinder similarly demonstrate that heat-transfer enhancement arises from the combined action of pulsation intensity, flow regime, and wake dynamics [9]. Duty cycle therefore warrants explicit consideration because it controls the proportion of each cycle during which the fluid experiences an active or high-flow condition.

Engineering evaluation of pulsating flow should distinguish between absolute heat-transfer performance and enhancement relative to steady flow. A high Nusselt number can result primarily from an increased Reynolds number, whereas a high Nu_p/Nu_{st} ratio indicates a stronger contribution from pulsation. Both measures are needed to avoid selecting a regime that performs well only because of a favourable background flow condition. Experimental investigations of pulsating flow in non-straight tubes have also shown that thermal enhancement may be accompanied by increased flow resistance [10]. Consequently, a thermal optimum should not automatically be described as a complete thermo-hydraulic optimum when pressure-drop, friction-factor, or pumping-power measurements are unavailable.

Computational analysis provides a practical means of extracting additional engineering knowledge from structured experimental data. Factorial modelling can isolate the effects of frequency, amplitude, and duty cycle after accounting for Reynolds and Prandtl numbers. Interaction terms can be used to determine if the effect of one pulsation factor is related to another. Uncertainty-weighted regression can help account for variation in measurement precision and robust standard errors can help safeguard inference against nonconstant residual variance. The statistical relationships can then be translated into dimensionless power-law correlations that can be used to derive equations that have a direct relationship with thermal-fluid physics.

Random train-test splitting is not adequate for repeated factorial experiments as observations under the same background flow condition are related. Observations from different subsets of the data (training vs. test) may lead to a biased estimate of predictive performance. Instead, validation should be performed in complete operating-condition blocks to test the predictions in Reynolds-Prandtl conditions different from the ones used to estimate the model. This approach gives a more challenging test of transferability in the range of the measured experimental domain.

The experimental data available is a well-balanced set of 180 regimes for an in-line tube bundle with cross flow of reciprocating pulsating flow. The frequency was tested at three values, the amplitude ratio at three values, and the duty cycle at two values for each of a total of 10 operating-condition blocks. Such a design allows for statistical separation of main effects, interactions, continuous flow co-variates and measurement uncertainty. It also allows for the direct comparison of all 18 pulsation settings under repeated Reynolds-Prandtl conditions.

The present investigation is an experimental study with computational modelling to provide characterization of the thermal-fluid response of the tube-bundle system. Experimental measurements are not just used to describe and compare them with other observations. They are employed to create uncertainty-aware factorial models, to verify the predictive performance in held-out operating conditions, to define dimensionless engineering correlations and to establish a robust operating regime. Conclusions are limited to the range of experimental pressure drop found in the experiment and do not imply performance of the observed pressure drop outside of the tested design space or global optimality.

Following objectives were established:

- To investigate the effects of Reynolds number, Prandtl number, pulsation frequency, amplitude ratio, and duty cycle on the Nusselt number and heat-transfer enhancement ratio of the in-line tube bundle.
- To develop and validate uncertainty-aware computational models and physically interpretable dimensionless correlations for predicting the fluid-flow and heat-transfer responses within the measured experimental domain.

- To identify the most effective experimentally tested pulsation regime and evaluate its reliability through blocked validation, bootstrap analysis, Pareto assessment, and decision-sensitivity testing.

2. LITERATURE REVIEW

The flow around bluff bodies is pulsating when the externally imposed unsteadiness interacts with the natural separation. The velocity fluctuations change stagnation-point transport, the boundary-layer thickness, the separation angle, the recirculation length, and the vortex-shedding frequency. The heat-transfer response to the imposed pulsation is found to be different depending on whether coherent mixing is enhanced or hindered prior to the development of vortical structures. The pulsation amplitude and frequency have been found to modify the drag, wake organization and Nusselt-number distribution around a heated semi-circular cylinder at low and moderate Reynolds-number conditions through numerical investigation [11]. This behavior is evidence that pulsation parameters should be considered not as independent sources of enhancement, but as interacting.

Computational modelling of flow past a circular cylinder has provided further insight into transient thermal transport. Lattice Boltzmann simulations have shown that pulsation changes the timing of vortex release and the thickness of the thermal boundary layer surrounding a constant-temperature cylinder [12]. Periodic acceleration increases local velocity gradients, while deceleration can enlarge the recirculation zone and promote interaction between heated near-wall fluid and cooler fluid from the mainstream. Time-averaged enhancement therefore reflects the cumulative effect of highly nonuniform heat transfer over the pulsation cycle.

Fluid rheology can also modify the response to pulsation. Numerical work involving shear-thinning power-law fluids around a confined semi-circular cylinder found that heat transfer was governed by the combined effects of Reynolds number, power-law index, pulsation frequency, and amplitude [13]. Although the present study concerns water, such evidence remains relevant because it demonstrates that pulsation cannot be interpreted independently of fluid transport properties. Inclusion of Prandtl number is therefore necessary when heat-transfer results span different mean fluid temperatures.

Tube bundles are different from isolated cylinders because the downstream tubes are influenced by the wake produced by the upstream tubes. In-line arrangements result in a series of flow passages with jet-like gap acceleration followed by wake expansion. These repeated disturbances are amplified or weakened by pulsation depending on the relationship between the convective travel time and the pulsation. A numerical study of pulsating nanofluid flow over cam-shaped tube bundles revealed that the Nusselt number and friction factor were influenced by the Womersley number, pulsation amplitude and Reynolds number [14]. A critical frequency was found, where further increase in the pulsation did not lead to further improvement of the thermal performance. This non-monotonic behavior is what makes it possible to use more than one of the tested frequencies and formal optimization, instead of assuming the highest frequency is always best.

Frequency is a measure of how fast the flow speeds up and slows down, and amplitude is a measure of the size of the velocity changes from cycle to cycle. Together, they control boundary-layer interruption, secondary-flow intensity, reverse flow and fluid mixing. The square-wave pulsation has faster velocity changes than sinusoidal pulsation and may produce higher transient shear. Numerical analysis of square-wave pulsating flow in a helical tube found that increasing amplitude and dimensionless frequency intensified secondary flow and turbulence but also increased flow resistance [15]. Maximum comprehensive performance occurred at an intermediate pulsation condition rather than at the most extreme parameter values. Geometry and waveform may act synergistically. Research combining sinusoidal pulsation with annular corrugation in a helical tube reported that pulsation modified the secondary-flow field and improved alignment between velocity and temperature gradients [16]. Nusselt number initially increased and subsequently decreased with increasing dimensionless frequency, whereas friction resistance continued to rise. Such findings reinforce two points relevant to tube-bundle analysis. First, thermal response may be nonlinear across frequency levels. Second, the setting that maximizes heat transfer may not maximize thermo-hydraulic efficiency.

Computational modelling of pulsating flow in a straight circular pipe has also shown that heat-transfer enhancement depends jointly on Reynolds number, frequency, and amplitude [17]. Flow reversals and repeated redevelopment of the thermal boundary layer can produce substantial enhancement under selected conditions. However, extrapolation beyond the modelled domain remains unsafe because changes in flow regime can modify the governing mechanism. Data-driven models should consequently be validated within clearly defined Reynolds, Prandtl, and pulsation ranges.

Waveform asymmetry introduces another level of control. Duty cycle is the time that the flow is spent in each part of the pulsation wave form. A shorter active-flow interval may create a steeper acceleration and better instantaneous mixing, while a longer interval may allow more time for convective transport, but allow partial boundary-layer redevelopment. The duty-cycle effects can also be dependent on amplitude and frequency and hence interaction terms are necessary in a factorial analysis. The literature has focused more on sinusoidal frequency and amplitude than on the asymmetry of the waveforms. This imbalance is a clear argument for an explicit consideration of duty cycle in the current study.

Analytical solutions are still useful when it comes to determining the physical parameters which control the pulsating convection. Research on pulsating flow in porous channels and tubes has shown how the interaction between the periodic forcing, the viscous resistance and the thermal diffusion affects the flow [18]. Although porous-medium conditions differ from tube-bundle crossflow, the underlying methodological principle is transferable. Dimensionless groups should be used to represent competing transport mechanisms, and their effects should be interpreted within the assumptions and limits of the system.

Turbulent pulsating flow presents additional difficulties because time-dependent forcing interacts with an already unsteady velocity field. Investigation of pulsating turbulent gas flow in a tube showed that frequency response and heat-transfer variation depend on the thermal inertia of the boundary layer and the structure of turbulent transport [19]. Correlations derived under one waveform or flow regime therefore cannot be transferred automatically to another. Empirical equations need to be tested against the withheld observations, and the coefficients of the equation should be given with uncertainty intervals.

The power law correlations are commonly used in the field of heat transfer engineering because they can compactly represent the relationship between Nu , Re and Pr and other dimensionless parameters. The expression can be generalized as $Nu = CRe^aPr^bX^c$, where X represents a geometry or pulsation parameter. The equations are still physically meaningful since each exponent represents an effect that is approximately equal to the direction and strength of the effect in question. However, the high coefficient of determination in the in-sample is not enough to show reliability. Correlations can be a good fit for repeated experimental data yet fail to be a good fit for an operating condition not observed during the experiment. It is therefore important to have blocked validation and bootstrap analysis to estimate predictive transferability and the coefficient stability. Recent study of a flat plate in pulsating duct flow with nonuniform inflow has also demonstrated that it is possible to enhance heat transfer by restructuring the flow upstream and to periodically renew the thermal boundary layer [20]. The effects were dependent upon the inflow condition and the characteristics of the pulsation, showing that enhancement is system dependent. This is especially applicable for an in-line tube bundle where each row gets a nonuniform inflow both in space and time, created by the previous rows.

The majority of the published works have been restricted to isolated cylinders, pipe flows, helical tubes, porous passages or modified heat transfer surfaces. The studies that have dealt with reciprocating pulsating cross-flow through an in-line tube bundle have not considered the frequency, amplitude ratio, duty cycle, Reynolds number, Prandtl number or measurement uncertainty. The mean Nusselt-number trends are often reported in existing work, but the factorial inference, uncertainty weighting, out-of-block prediction, development of dimensionless correlation and decision-robust optimization are not performed in one analytical framework.

Only a little attention has been paid to the difference between absolute Nusselt number and relative enhancement, either. The Reynolds number might be the more important parameter in determining the absolute Nu , and the pulsation parameters might be more important in determining Nu_p/Nu_{st} . When assessing just one response, it is possible to get a partial interpretation. This requires joint analysis to determine if an operating condition will be high total heat transfer or strong improvement over steady flow or both.

Another methodological gap is around predictive validation. In-sample/Randomly Partitioned statistics are often used for models that are fitted to repeated factorial data. These do not provide sufficient evaluation of the ability of a model to transfer to different background flow conditions. Completely omitting Reynolds-Prandtl blocks provides a more realistic evaluation as there is no observation from the missing operating condition that helps to fit the model.

In this work, we tackle these limitations by combining experimental measurements and uncertainty-aware statistical and computational techniques. The simultaneous modelling of Nu and Nu_p/Nu_{st} , the estimation of factorial main and interaction effects, the comparison of OLS-HC3 results and inverse-variance WLS results, the leave-one-block-out prediction, the block-bootstrap dimensionless correlations and the robust selection among the 18 measured pulsation settings are all included in the contributions. Scope remains intentionally bound to thermal performance because the available data do not include pressure drop, friction factor, or pumping power.

3. METHODOLOGY

3.1 Research design and analytical framework

A quantitative computational design was used to investigate fluid-flow and heat-transfer behavior in an in-line tube bundle exposed to reciprocating pulsating crossflow. Experimental measurements produced by Hayrullin et al. formed the empirical basis of the study [21]. Consequently, the experimental component involved secondary analysis of an independently generated laboratory dataset. No new physical experiments were conducted by the present authors. Computational work covered data-quality auditing, uncertainty-aware statistical inference, factorial modelling, blocked predictive validation, development of dimensionless engineering correlations, and optimization of the measured operating regimes.

Two response variables were examined. Nusselt number, Nu , was used to represent absolute convective heat-transfer performance. Heat-transfer enhancement ratio, Nu_p/Nu_{st} , was used to express the performance of pulsating flow relative to the corresponding steady-flow condition. Fluid-flow and pulsation characteristics were represented by gap velocity, Reynolds number, Prandtl number, Strouhal number, Fourier number, pulsation frequency, amplitude ratio, and duty cycle. This structure allowed the analysis to distinguish changes in absolute heat transfer from changes attributable specifically to pulsation.

3.2 Dataset, physical variables and preprocessing

An open experimental dataset entitled Experimental study of heat transfer in an in-line tube bundle under reciprocating pulsating cross-flow: effects of Prandtl number and waveform asymmetry was obtained from Mendeley Data, Version 1 [21]. Laboratory measurements in the dataset describe water flow across an in-line tube bundle under reciprocating pulsation. A total of 180 observations and 22 original variables were available. Each observation represented one measured operating regime, and corresponding uncertainties were provided for the principal dimensional, dimensionless, and heat-transfer variables.

Experimental factors followed a balanced $3 \times 3 \times 2$ arrangement. Pulsation frequency was tested at 0.180, 0.315, and 0.450 Hz. Amplitude ratio, A/D , was tested at 5, 10, and 15. Duty cycle, ψ , was tested at 0.2 and 0.5. These levels produced 18 distinct pulsation settings. Each setting was observed within ten Reynolds-Prandtl operating-condition blocks, giving ten observations per factorial combination and 180 observations overall. Reynolds numbers ranged from 1191 to 3400, while Prandtl numbers ranged from 3.49 to 5.03. Nusselt numbers ranged from 32.4 to 93.0, and enhancement ratios ranged from 0.90 to 1.66.

Reynolds number was used to describe the relationship between inertial and viscous forces:

$$Re = \frac{\rho u_{\text{gap}} D}{\mu} \quad (1)$$

where ρ is fluid density, u_{gap} is the gap velocity, D is the characteristic tube diameter, and μ is dynamic viscosity.

Convective heat-transfer performance was represented by the Nusselt number:

$$Nu = \frac{hD}{k} \quad (2)$$

where h is the convective heat-transfer coefficient and k is fluid thermal conductivity. Relative improvement under pulsating flow was calculated as:

$$E_{Nu} = \frac{Nu_p}{Nu_{st}} \quad (3)$$

where Nu_p is the Nusselt number measured under pulsating flow and Nu_{st} is its steady-flow reference. A value greater than one represents heat-transfer enhancement, a value equal to one indicates no change, and a value below one represents deterioration relative to steady flow.

Strouhal number characterized the relationship between pulsation frequency and convective transport:

$$St = \frac{fD}{u_{\text{gap}}} \quad (4)$$

where f is pulsation frequency. A combined pulsation parameter was also considered:

$$\Pi_p = \left(\frac{A}{D}\right) St \quad (5)$$

where A/D is the dimensionless amplitude ratio. Fourier number represented thermal diffusion relative to the pulsation timescale, while duty cycle described waveform asymmetry.

Variable names were standardized using explicit engineering terminology and units. Regime identifiers were retained for traceability but excluded from modelling. Data quality was assessed through checks for missing values, duplicate rows, duplicate regime identifiers, nonnumeric entries, nonpositive dimensionless quantities, and observations outside physically credible ranges. No missing values, duplicates, or invalid measurements were detected. All 180 observations were retained.

Consistency of the reported combined pulsation parameter was evaluated by comparing it with the reconstructed product of amplitude ratio and Strouhal number:

$$\varepsilon_{\Pi} = \left| \Pi_{p,\text{reported}} - \left(\frac{A}{D}\right) St \right| \quad (6)$$

Maximum absolute disagreement was 0.0005, which was sufficiently small to be attributed to numerical rounding. Relative uncertainty for each measurement-uncertainty pair was calculated as:

$$U_{r,i}(\%) = \frac{U_i}{|x_i|} \times 100 \quad (7)$$

where x_i is the measured value and U_i is its reported expanded uncertainty. No observation was removed because of comparatively high uncertainty. Instead, uncertainty was incorporated explicitly through inverse-variance weighting and block-bootstrap analysis.

3.3 Statistical analysis and factorial modelling

Descriptive analysis was performed before inferential modelling. Continuous variables were summarized using sample size, mean, standard deviation, standard error, 95% confidence interval, median, first and third quartiles, minimum, maximum, coefficient of variation, skewness, and excess kurtosis. Mean Nusselt number and mean enhancement ratio were also calculated separately for each frequency, amplitude ratio, and duty-cycle level.

Uncertainty-weighted summaries were calculated using inverse-variance weighting:

$$\bar{y}_w = \frac{\sum_{i=1}^n w_i y_i}{\sum_{i=1}^n w_i}, \quad w_i = \frac{1}{U_{y,i}^2} \quad (8)$$

where y_i is the measured response and $U_{y,i}$ is its reported uncertainty. This approach gave greater statistical weight to more precise measurements without excluding observations with larger uncertainty.

Linear relationships were evaluated by Pearson correlation coefficient, and monotonic relationship was evaluated by Spearman rank correlation. Both measures were investigated since some dimensionless variables had skewed distributions and/or nonlinear dependence. To control for the proportion of false positive results in the outcome-predictor tests, Benjamini-Hochberg false-discovery-rate correction was used.

The variance inflation factors and pairwise correlations were used to check for multicollinearity. The correlation between the temperature and Prandtl number, gap velocity and Reynolds number, and frequency and Fourier number were strong. Strouhal number and the combined pulsation parameter were also mathematically correlated with some primary variables. Strouhal

number and combined pulsation parameter were also mathematically related with some primary variables. For this reason, direct mathematical derivatives of raw measurements were not recorded simultaneously with the raw measurements in the main factorial models. The Reynolds number, Prandtl number, frequency, amplitude ratio and duty cycle were kept as they were physically interpretable and statistically stable representation of the experimental design.

Reynolds and Prandtl numbers were standardized before model fitting:

$$Re_z = \frac{Re - \bar{Re}}{s_{Re}} \quad (9)$$

$$Pr_z = \frac{Pr - \bar{Pr}}{s_{Pr}} \quad (10)$$

where $\bar{Re}$ and $\bar{Pr}$ are the sample means, while s_{Re} and s_{Pr} are the corresponding sample standard deviations. Standardization placed the two continuous covariates on comparable scales and allowed their coefficients to be interpreted as changes associated with a one-standard-deviation increase.

Separate models were fitted for Nusselt number and heat-transfer enhancement ratio. Frequency, amplitude ratio, and duty cycle were treated as categorical experimental factors because they were tested at fixed levels. Reynolds and Prandtl numbers were treated as continuous covariates. A full factorial analysis-of-covariance model was specified as:

$$y = \beta_0 + \beta_1 Re_z + \beta_2 Pr_z + F + A + \psi + F:A + F:\psi + A:\psi + F:A:\psi + \varepsilon \quad (11)$$

where y represents either Nu or E_{Nu} , F represents pulsation frequency, A represents amplitude ratio, and ψ represents duty cycle. Two-way interaction terms evaluated whether the effect of one pulsation parameter depended on another. The three-way interaction tested whether the combined frequency-amplitude response changed across duty-cycle conditions. Sum-to-zero contrasts were applied so that factor coefficients represented deviations from the grand mean and Type III tests remained interpretable in the presence of interactions.

Ordinarily, at least squares estimation was used to quantify factorial effects. Heteroscedasticity-consistent HC3 standard errors were used for coefficient-level inference because constant residual variance could not be assumed. Weighted least squares models were fitted as an uncertainty-aware robustness analysis using:

$$w_{i,Nu} = \frac{1}{U_{Nu,i}^2} \quad (12)$$

and

$$w_{i,E} = \frac{1}{U_{E,i}^2} \quad (13)$$

where $U_{Nu,i}$ and $U_{E,i}$ are the reported uncertainties for Nusselt number and enhancement ratio, respectively.

Type III ANCOVA was used to test main effects and interactions after adjustment for Reynolds and Prandtl numbers. Statistical significance was evaluated at $\alpha = 0.05$. Effect magnitude was quantified using partial eta squared:

$$\eta_p^2 = \frac{SS_{\text{effect}}}{SS_{\text{effect}} + SS_{\text{error}}} \quad (14)$$

where SS_{effect} is the sum of squares associated with a tested term and SS_{error} is the residual sum of squares. Effect sizes were interpreted alongside p-values so that practically important effects were distinguished from effects that were statistically significant mainly because of low residual variation.

Model adequacy was assessed using R^2 , adjusted R^2 , root-mean-square error, mean absolute error, residual standard deviation, Shapiro-Wilk residual analysis, the Breusch-Pagan heteroscedasticity test, Durbin-Watson statistic, leverage, and Cook's distance. Agreement between OLS-HC3 and weighted estimates was treated as evidence that the main conclusions were not dependent on a single variance assumption.

3.4 Predictive validation and dimensionless correlations

Random observation-level cross-validation was avoided because observations recorded within the same Reynolds-Prandtl condition formed a related operating block. Randomly distributing these observations between training and test sets could allow closely related regimes to occur in both sets and produce an overly optimistic estimate of prediction accuracy.

Leave-one-block-out validation was therefore applied. Ten operating-condition blocks were reconstructed, with each block containing all 18 frequency-amplitude-duty-cycle combinations. During each outer fold, one complete block was withheld. Models were fitted using the remaining nine blocks and then used to predict the 18 observations in the omitted block. Repeating this process across all ten blocks produced 180 out-of-fold predictions for each outcome.

Additive, two-way-interaction, and full three-way-interaction models were compared under both OLS and WLS estimation. Predictive performance was quantified using root-mean-square error:

$$RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^n (y_i - \hat{y}_i)^2} \quad (15)$$

mean absolute error:

$$\text{MAE} = \frac{1}{n} \sum_{i=1}^n |y_i - \hat{y}_i| \quad (16)$$

mean absolute percentage error:

$$\text{MAPE} = \frac{100}{n} \sum_{i=1}^n \left| \frac{y_i - \hat{y}_i}{y_i} \right| \quad (17)$$

and out-of-fold coefficient of determination:

$$R_{\text{OOF}}^2 = 1 - \frac{\sum_{i=1}^n (y_i - \hat{y}_i)^2}{\sum_{i=1}^n (y_i - \bar{y})^2} \quad (18)$$

where y_i is the observed response, $\hat{y}_i$ is, its out-of-fold prediction, and $\bar{y}$ is the overall observed mean. Normalized RMSE was additionally expressed relative to the observed response range and mean. Final models were selected according to blocked predictive accuracy, physical interpretability, and parsimony rather than in-sample fit alone.

Physics-informed dimensionless correlations were developed for Nu and E_{Nu} . Candidate equations were expressed in a general multiplicative form:

$$y = C \prod_{j=1}^p X_j^{b_j} \quad (19)$$

where C is a fitted constant, X_j is a dimensionless predictor, and b_j is its estimated exponent. Logarithmic transformation produced the linear estimation form:

$$\ln(y) = \ln(C) + \sum_{j=1}^p b_j \ln(X_j) + \varepsilon \quad (20)$$

Five candidate formulations were compared: a classical Reynolds-Prandtl form, a direct-pulsation form, a Strouhal-based form, a Fourier-based form, and a combined-pulsation-parameter form. Candidate predictors included Re , Pr , St , Fo , A/D , ψ , and Π_p . Mathematically redundant terms were not combined when their simultaneous use produced severe multicollinearity or prevented a clear physical interpretation.

Blocked prediction error was used to compare the candidate correlations. A parsimonious model was preferred when its RMSE was within 2% of the minimum candidate RMSE and it used fewer or more physically interpretable terms. Coefficient stability was examined through 2000 block-bootstrap replications. Entire Reynolds-Prandtl blocks were resampled with replacement, preserving the complete factorial structure within each block. Bootstrap means standard deviations, percentile-based 95% confidence intervals, and exponent-sign stability were calculated for every retained coefficient.

3.5 Operating-regime optimization and reproducibility

Mean Nu and E_{Nu} were calculated for each of the 18 experimentally observed pulsation settings. Individual response desirability was transformed to a scale from zero to one:

$$d_k(x) = \frac{y_k(x) - y_{k,\min}}{y_{k,\max} - y_{k,\min}} \quad (21)$$

where $d_k(x)$ is the desirability of response k at operating setting x . A value of zero represented the least favourable observed response, while a value of one represented the most favourable observed response.

Joint thermal desirability was calculated using a weighted geometric mean:

$$D(x) = [d_{Nu}(x)^{w_1} d_{E_{Nu}}(x)^{w_2}]^{\frac{1}{w_1 + w_2}} \quad (22)$$

where w_1 and w_2 represent the weights assigned to Nusselt number and enhancement ratio. Equal weights were used in the primary analysis. Pareto analysis was also performed to identify nondominated operating settings for which neither thermal outcome could be improved without reducing the other.

Bootstrap resampling was used to estimate the probability that each experimental setting was optimal. Pairwise superiority of the selected regime over competing settings was evaluated using block-bootstrap confidence intervals, paired t-tests, and Wilcoxon signed-rank tests. Holm adjustment controlled the family-wise error rate across the multiple setting comparisons.

Nested leave-one-block-out optimization was conducted to assess whether the selected regime remained effective under unseen flow conditions. During each fold, an optimum was selected using only the nine training blocks. Its observed performance was then evaluated in the withheld block and compared with the corresponding low-pulsation reference and the best setting observed within that block. Robustness was summarized using optimum-selection frequency, number of positive-improvement blocks, mean held-out gain, bootstrap confidence intervals, probability of positive improvement, and desirability regret.

Decision sensitivity was examined across 189 scenarios generated by varying the relative weights assigned to Nu and E_{Nu} , response-scaling assumptions, and uncertainty conditions. Rank stability and the desirability margin between the selected regime and the second-ranked setting were calculated. A setting was considered decision-robust only when it remained optimal across blocked validation, bootstrap resampling, and sensitivity scenarios.

Optimization was restricted to the 18 measured combinations. No continuous or global optimum beyond the observed frequency, amplitude ratio, duty cycle, Reynolds-number, or Prandtl-number ranges were inferred. Complete thermo-hydraulic

optimization was also not claimed because pressure drop, friction factor, and pumping power were unavailable. Accordingly, the identified optimum represents the strongest experimentally supported thermal setting within the tested domain. All computations were performed in Python 3.10 using NumPy, pandas, SciPy, statsmodels, scikit-learn, Matplotlib, and seaborn. Fixed random seeds were used for bootstrap and resampling procedures. Cleaned data, fitted models, validation predictions, statistical tables, figures, and optimization outputs were saved through sequential computational checkpoints. This structure allowed every reported result to be traced to the source dataset and reproduced without undocumented manual modification.

4. RESULTS

4.1 Data integrity and experimental-domain characteristics

All 180 experimental observations were found to be suitable for analysis when checked for quality. There were no missing values, duplicated records, duplicated regime identifiers, non-numeric variables, or non-positive uncertainty estimates found. All measurements were within physically reasonable limits. Gap velocity varied from 0.0648 to 0.1850 m/s, Reynolds number from 1,191 to 3,400, Prandtl number from 3.49 to 5.03, and Strouhal number from 0.00973 to 0.06890. The range of Fourier number was 0.00331 to 0.00871, and the range of the combined pulsation parameter $(A/D) St$ was 0.049 to 1.034. The mean absolute error and the maximum error of the reconstruction of $(A/D) St$ from its constituent variables are 0.000204 and 0.000500, respectively, indicating internal consistency after numerical rounding.

The experimental design was fully balanced. Three frequency levels, 0.180, 0.315, and 0.450 Hz, were combined with amplitude ratios of 5, 10, and 15 and duty cycles of 0.2 and 0.5. Ten Reynolds–Prandtl operating blocks were used to represent each of the 18 factorial combinations. Therefore, unequal sample sizes were not a problem when comparing the different settings of the pulsation control.

The quality of the measurements was also satisfactory. The mean relative uncertainties were 2.02% for gap velocity, 2.20% for Reynolds number, 2.24% for Prandtl number, 4.43% for Nusselt number and 6.25% for the enhancement ratio. This relative uncertainty in the combined pulsation parameter was as high as 20.41% at its lowest values, but the median uncertainty was only 2.47%. This pattern suggested that the higher percentage was not due to measurement instability but was mostly due to the small values of the parameters.

The mean Nusselt number was 58.869 with a standard deviation of 12.323 and a range of 32.4–93.0. The average enhancement ratio of mean heat transfer was 1.247, which means that the average improvement of the steady-flow Nusselt number was about 24.7%. In 172 of the 177 regimes, enhancement was greater than unity, which is 95.56% of the data set. Ratios of at least 1.10, 1.25, and 1.50 were observed at 81.67%, 47.22%, and 8.33% of the regimes, respectively.

The mean outcomes were found to be increasing in a progressive manner with increasing pulsation frequency and increasing ratio of amplitude. The mean Nusselt number increased from 52.625 to 64.893, corresponding to an increase of 23.3% as the frequency was increased from 0.180 to 0.450 Hz. Mean enhancement simultaneously increased from 1.113 to 1.377, corresponding to 23.7%. Increasing A/D from 5 to 15 raised mean Nusselt number from 53.703 to 63.768 and enhancement from 1.135 to 1.353. Duty cycle exhibited the opposite directional pattern: $\psi = 0.2$ produced higher mean Nusselt number and enhancement than $\psi = 0.5$, as summarized in Table 1.

Table 1. Descriptive statistics and thermal outcomes across the experimental factor levels

Variable or operating level	N	Mean	SD	Minimum	Maximum
Reynolds number	180	1,959.289	669.632	1,191	3,400
Prandtl number	180	4.250	0.643	3.490	5.030
Gap velocity (m/s)	180	0.1248	0.0395	0.0648	0.1850
Strouhal number	180	0.0288	0.0148	0.0097	0.0689
Combined pulsation parameter	180	0.2880	0.1990	0.0490	1.0340
Nusselt number	180	58.869	12.323	32.400	93.000
Enhancement ratio	180	1.247	0.162	0.900	1.660
Frequency = 0.180 Hz: Nusselt number	60	52.625	10.295	32.400	75.000
Frequency = 0.315 Hz: Nusselt number	60	59.088	11.206	36.100	84.000
Frequency = 0.450 Hz: Nusselt number	60	64.893	12.351	40.200	93.000
Frequency = 0.180 Hz: enhancement	60	1.113	0.096	0.900	1.340
Frequency = 0.315 Hz: enhancement	60	1.252	0.120	1.010	1.490
Frequency = 0.450 Hz: enhancement	60	1.377	0.142	1.130	1.660
$A/D = 5$: Nusselt number	60	53.703	10.916	32.400	78.000
$A/D = 10$: Nusselt number	60	59.135	11.468	36.100	85.000
$A/D = 15$: Nusselt number	60	63.768	12.586	39.400	93.000
$A/D = 5$: enhancement	60	1.135	0.105	0.900	1.350
$A/D = 10$: enhancement	60	1.254	0.140	1.000	1.590
$A/D = 15$: enhancement	60	1.353	0.156	1.060	1.660
Duty cycle = 0.2: Nusselt number	90	60.019	12.553	33.000	93.000
Duty cycle = 0.5: Nusselt number	90	57.719	12.048	32.400	90.000

Duty cycle = 0.2: enhancement	90	1.269	0.151	1.020	1.660
Duty cycle = 0.5: enhancement	90	1.225	0.170	0.900	1.660

Table 1 demonstrates that both flow intensity and pulsation control contributed to the measured thermal response. Reynolds number primarily governed absolute convective transfer, whereas frequency and amplitude ratio produced pronounced improvements relative to steady flow. Lower duty cycle provided a smaller but consistent additional benefit.

Response maps provide a more direct engineering interpretation of these results. At both duty cycles, increasing frequency moved the response toward higher Nusselt and enhancement regions. Increasing amplitude produced a similar progression, with the strongest response consistently located at 0.450 Hz and A/D = 15. No reversal of this trend occurred within the tested control domain, as shown in Figure 1.

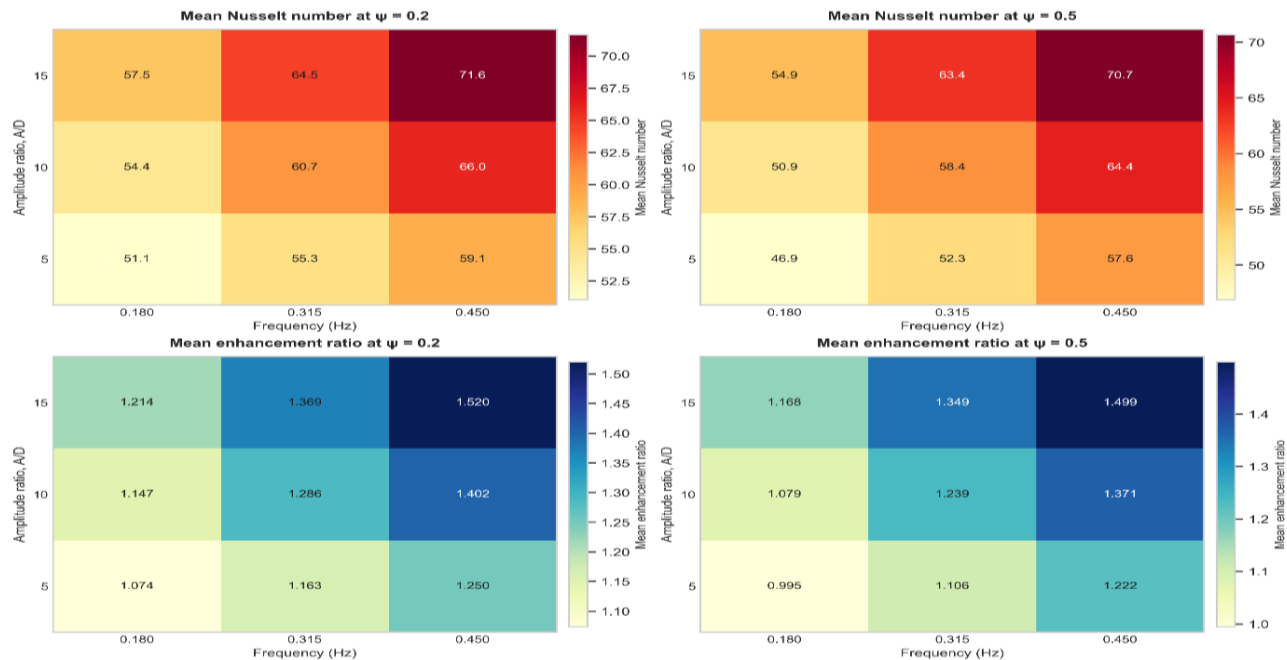


Figure 1. Experimental response maps of mean Nusselt number and heat-transfer enhancement ratio across pulsation frequency, amplitude ratio, and duty cycle.

Figure 1 shows a coherent frequency–amplitude response rather than isolated improvements at individual settings. Higher frequency and amplitude jointly intensified the thermal response, while $\psi = 0.2$ generally shifted comparable cells toward higher values. This pattern directly satisfies the objective of characterizing heat-transfer behaviour across controllable pulsating-flow conditions.

4.2 Dependence structure and factorial effects

Correlation analysis distinguished variables associated with absolute heat transfer from those governing relative enhancement. Nusselt number correlated strongly with gap velocity ($r = 0.822$) and Reynolds number ($r = 0.773$), confirming the dominant role of bulk flow intensity. Moderate positive relationships were found with frequency ($r = 0.408$) and amplitude ratio ($r = 0.334$). Enhancement behaved differently. It had the highest correlation with the combined pulsation parameter ($r = 0.898$), the Strouhal number ($r = 0.710$), the frequency ($r = 0.669$) and the amplitude ratio ($r = 0.552$). Enhancement was found to be correlated negatively with Reynolds number ($r = -0.326$), and it was found that the absolute Nusselt number increased with the flow intensity, but the proportional increase in Nusselt number due to pulsation was greater at lower Reynolds numbers.

Multicollinearity diagnostics were used and it was determined that Reynolds number, Prandtl number, frequency, amplitude ratio, and duty cycle were the most appropriate independent variables. This specification had variance inflation factors between 1.000 and 1.085. The factorials regression was not performed for the derived quantities, as they were highly correlated with the other quantities. This variable selection method avoided the duplication of the same physical information.

The adjusted effects of pulsation controls were quantified by Type III factorial ANCOVA, after the effect of Reynolds and Prandtl numbers were removed. Both Nusselt number and enhancement were highly influenced by frequency, amplitude ratio and duty cycle. Both outcomes were also found to be influenced by frequency–amplitude interaction: an increase in the amplitude of oscillation was beneficial when the pulsation frequency was high. Three-way interaction was negligible, showing that a more complex combined effect was not required to explain the measured response (Table 2).

Table 2. Type III ANCOVA results for the two thermal outcomes

Outcome	Effect	F-value	p-value	Partial η^2	Magnitude
Nusselt number	Frequency	905.739	5.575×10^{-88}	0.9188	Large

Nusselt number	Amplitude ratio	605.313	2.377×10^{-75}	0.8833	Large
Nusselt number	Duty cycle	94.790	6.991×10^{-18}	0.3720	Large
Nusselt number	Frequency $\times$ amplitude ratio	15.955	5.184×10^{-11}	0.2851	Large
Nusselt number	Frequency $\times$ duty cycle	6.541	0.00186	0.0756	Medium
Nusselt number	Amplitude $\times$ duty cycle	2.498	0.0855	0.0303	Small, nonsignificant
Nusselt number	Three-way interaction	0.243	0.9136	0.0060	Negligible
Nusselt number	Reynolds number	7,433.796	4.973×10^{-136}	0.9789	Large
Nusselt number	Prandtl number	1,017.872	2.966×10^{-71}	0.8642	Large
Enhancement ratio	Frequency	376.915	2.887×10^{-61}	0.8249	Large
Enhancement ratio	Amplitude ratio	257.411	9.874×10^{-51}	0.7629	Large
Enhancement ratio	Duty cycle	31.478	8.684×10^{-8}	0.1644	Large
Enhancement ratio	Frequency $\times$ amplitude ratio	6.355	9.020×10^{-5}	0.1371	Medium
Enhancement ratio	Frequency $\times$ duty cycle	1.937	0.1475	0.0236	Small, nonsignificant
Enhancement ratio	Amplitude $\times$ duty cycle	0.987	0.3750	0.0122	Small, nonsignificant
Enhancement ratio	Three-way interaction	0.121	0.9749	0.0030	Negligible
Enhancement ratio	Reynolds number	190.725	4.587×10^{-29}	0.5438	Large
Enhancement ratio	Prandtl number	11.832	7.428×10^{-4}	0.0689	Medium

Table 2 establishes that absolute convection and pulsation-induced improvement are governed by related but nonidentical mechanisms. Reynolds and Prandtl numbers explained most variation in absolute Nu, while frequency and amplitude were the strongest controllable determinants of enhancement. Significant frequency–amplitude interactions support coordinated adjustment of these controls rather than independent tuning.

HC3-robust estimation produced positive standardized coefficients of 10.623 for Reynolds number and 3.931 for Prandtl number in the Nusselt model. Corresponding enhancement coefficients were -0.0568 and -0.0141 . Similar estimates from inverse-variance weighted regression confirmed that measurement uncertainty did not materially alter the principal conclusions. Nusselt residuals were approximately normal, but heteroscedasticity was detected; HC3 inference was therefore retained. Enhancement residuals showed no significant heteroscedasticity, supporting the adequacy of the fitted factorial structure.

4.3 Computational prediction and dimensionless correlation development

In-sample factorial models explained 98.53% of Nusselt variation and 90.49% of enhancement variation. More importantly, leave-one-operating-block-out validation assessed performance under Reynolds–Prandtl conditions excluded during model fitting. The two-way interaction model achieved a blocked R^2 of 0.9779 for Nusselt number, with RMSE of 1.826 and MAPE of 2.556%. Enhancement prediction achieved $R^2 = 0.7782$, RMSE = 0.0760, and MAPE = 4.760%. Its lower R^2 reflected the comparatively narrow enhancement range rather than large percentage error.

Physics-informed power-law modelling identified the Strouhal formulation as the best correlation structure for both outcomes. It outperformed classical Re–Pr equations and alternative formulations based on direct frequency, Fourier number, or the combined pulsation parameter. Resulting equations incorporated flow regime, fluid properties, pulsation timescale, displacement amplitude, and duty cycle. Their blocked-validation metrics are reported in Table 3.

Table 3. Predictive performance and derived dimensionless correlations

Outcome and model	Blocked R^2	RMSE	MAE	MAPE (%)	Model or equation
Nusselt factorial model	0.9779	1.8259	1.3992	2.556	Two-way interaction OLS
Enhancement factorial model	0.7782	0.0760	0.0591	4.760	Uncertainty-weighted factorial model
Nusselt correlation	0.9593	2.4790	1.9713	3.387	$\text{Nu} = 0.107295 \text{Re}^{0.775254} \text{Pr}^{0.627751} \text{St}^{0.228169} (\text{A}/\text{D})^{0.155304} \psi^{-0.041966}$
Enhancement correlation	0.8431	0.0639	0.0502	4.059	$\frac{\text{Nu}_p}{\text{Nu}_{\text{st}}} = 0.757998 \text{Re}^{0.100375} \text{Pr}^{0.132223} \text{St}^{0.230136} (\text{A}/\text{D})^{0.155786} \psi^{-0.041660}$

Table 3 shows that the factorial model offered the most accurate prediction of absolute Nusselt number, while the dimensionless correlations provided a compact and physically interpretable alternative. MAPE remained below 5% for every selected model, satisfying the computational objective of developing reliable predictive relationships within the experimental domain.

Exponent magnitudes provide additional physical insight. Reynolds number had the largest exponent in the Nusselt equation, followed by Prandtl and Strouhal numbers. Positive Stand A/D exponents indicated improved heat transfer with increasing pulsation frequency and displacement amplitude. Negative duty-cycle exponents supported the observed benefit of the shorter active-cycle condition. Block-bootstrap confidence intervals confirmed stable signs for all Nusselt exponents. Enhancement-

model Pruncertainty crossed zero, so its isolated influence remains less certain than those of St, A/D, duty cycle, and Reynolds number.

Observed-versus-predicted relationships clustered closely around the line of perfect agreement. Residual patterns showed limited systematic bias, and full-data plus blocked-validation performance remained reasonably close, as illustrated in Figure 2.

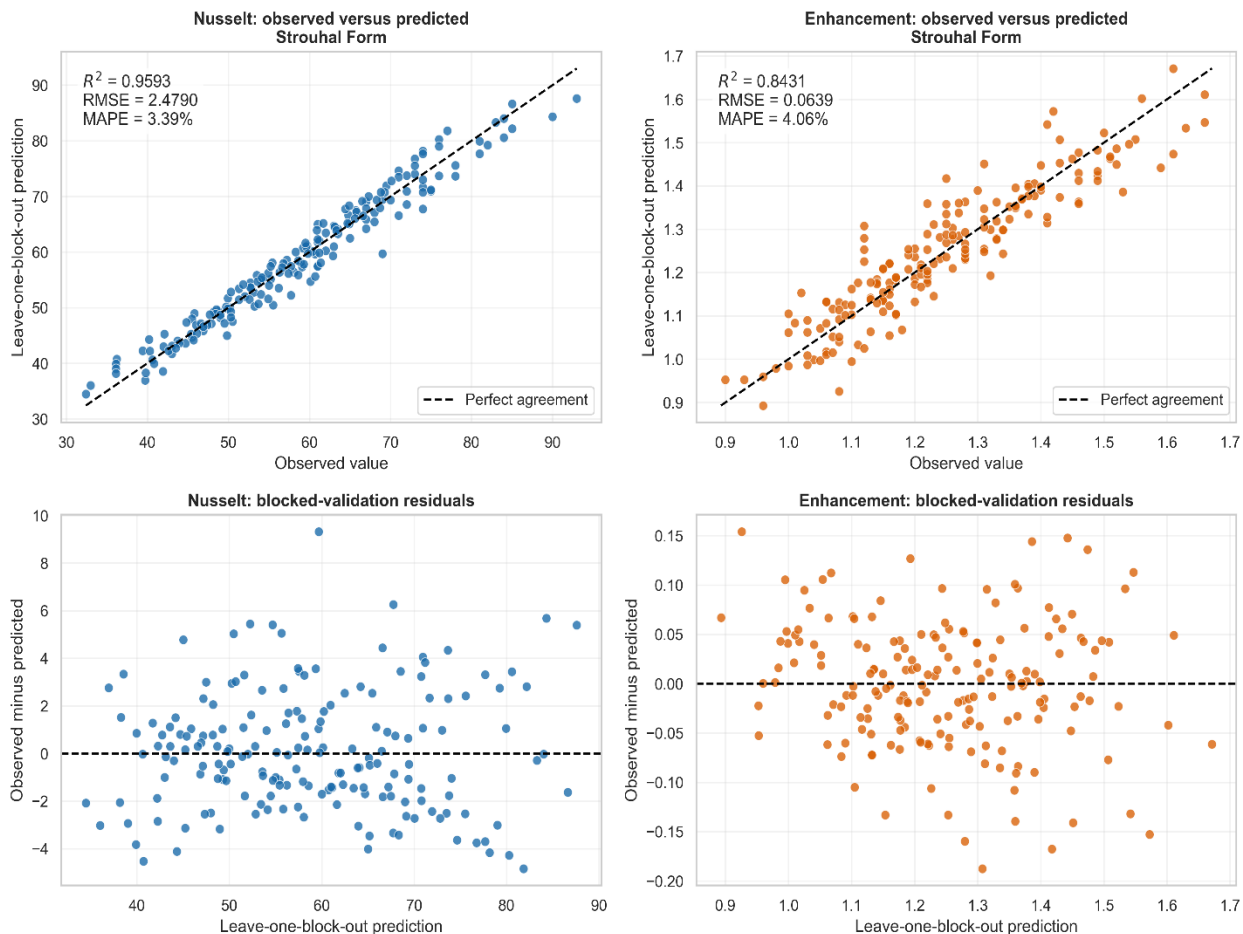


Figure 2. Blocked validation of the dimensionless Nusselt-number and heat-transfer-enhancement correlations.

Figure 2 confirms that predictive accuracy was retained when complete Reynolds–Prandtl blocks were excluded from model fitting. This evidence is stronger than random row-wise validation because it evaluates transfer to distinct operating conditions while preserving the factorial control structure.

4.4 Observed-domain thermal optimization

Uncertainty-adjusted multi-objective analysis evaluated all 18 experimental combinations using Nusselt number and enhancement ratio simultaneously. A single condition, $f = 0.450\text{Hz}$, $A/D = 15$, and $\psi = 0.2$, maximized both outcomes. Because the same setting optimized absolute transfer and proportional enhancement, no Pareto conflict occurred within the measured control domain.

Mean Nusselt number at the selected condition was 71.630, with a block-bootstrap 95% interval of 64.119–79.590. Mean enhancement ratio was 1.520, with an interval of 1.470–1.573. Compared with the mildest pulsation reference condition of 0.180 Hz, $A/D = 5$, and $\psi = 0.5$, the optimal setting increased Nu by 24.730 units and enhancement by 0.525. Relative improvements were 52.73% and 52.76%, respectively (Table 4).

Table 4. Statistical and validation evidence for the selected operating condition

Criterion	Nusselt number	Enhancement ratio
Optimal-setting mean	71.630	1.520
Optimal-setting bootstrap 95% CI	64.119–79.590	1.470–1.573
Reference-setting mean	46.900	0.995
Mean paired improvement	24.730	0.525
Relative improvement (%)	52.73	52.76
Paired-improvement bootstrap 95% CI	21.950–27.560	0.501–0.552
Positive improvement blocks	10 of 10	10 of 10
Bootstrap probability of positive improvement (%)	100.00	100.00
One-sided paired t-test p-value	2.624×10^{-8}	1.530×10^{-11}

One-sided Wilcoxon p-value	9.766×10^{-4}	9.766×10^{-4}
Probability of being best (%)	99.43	99.95
Outer-fold selection stability (%)	100.00	100.00

Table 4 demonstrates that the optimal setting was supported by effect magnitude, uncertainty intervals, parametric testing, nonparametric testing, and out-of-block validation. Superiority was therefore not dependent on a single statistical method or isolated high-performing observation.

Comparison against every alternative produced positive paired difference for both outcomes. All 17 Nusselt comparisons and all 17 enhancement comparisons retained bootstrap intervals above zero and remained significant after Holm adjustment. Bootstrap selection probabilities of 99.43% and 99.95% further indicated that the ranking was insensitive to resampling of the ten operating blocks.

Nested validation provided an additional safeguard against selection optimism. In every outer fold, the optimum was selected using only nine blocks and evaluated on the excluded tenth block. All ten excluded blocks produced positive gains. Mean held-out percentage improvements were 52.81% for Nusselt number and 52.88% for enhancement. Performance profiles for the selected and reference conditions are shown in Figure 3.

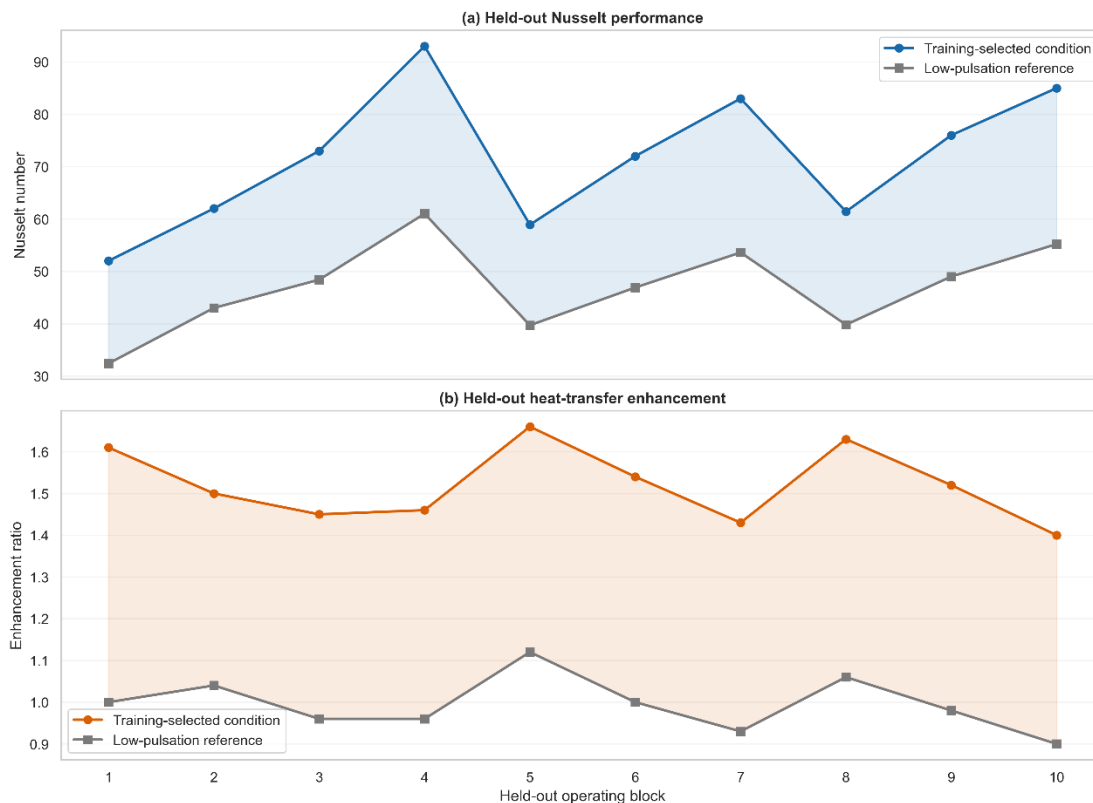


Figure 3. Out-of-block comparison of the training-selected operating condition and low-pulsation experimental reference.

Figure 3 shows that the thermal advantage persisted across low, intermediate, and high Reynolds–Prandtl blocks. Consistent separation between the selected and reference profiles indicates that the recommendation was not restricted to a narrow flow regime.

4.5 Decision robustness and objective fulfilment

Engineering decisions may depend on whether priority is assigned to absolute Nusselt number, proportional enhancement, or measurement conservatism. Sensitivity analysis therefore examined 21 outcome-weight combinations and nine uncertainty penalties, producing 189 scenarios. Every scenario selected $f = 0.450$ Hz, $A/D = 15$, and $\psi = 0.2$. Its rank remained exactly 1, with a desirability of 1.000 across the complete grid. The condition at the same frequency and amplitude but $\psi = 0.5$ remained second, with mean desirability of 0.96668. Minimum, mean, and maximum desirability margins over the runner-up were 0.01529, 0.03314, and 0.04000, respectively.

Nested decision regret was also minimal. Mean held-out desirability was 0.81295 compared with an oracle value of 0.81746. Mean regret was 0.00452, median regret was zero, and eight of ten folds selected the exact held-out oracle. Maximum regret was limited to 0.03422. Stability across outcome priorities and uncertainty penalties is presented in Figure 4.

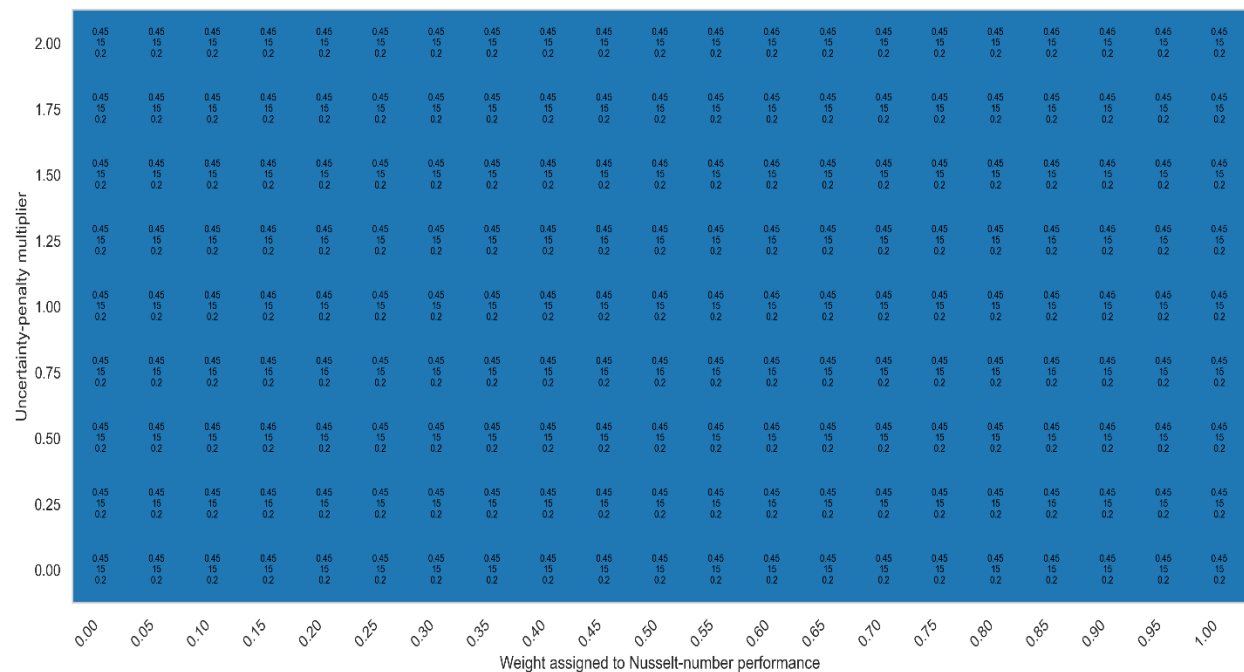


Figure 4. Sensitivity of the selected operating condition to Nusselt–enhancement weights and uncertainty-penalty multipliers.

Figure 4 establishes that the recommendation was independent of subjective weighting between the two thermal objectives. Robustness also persisted after subtracting up to twice the reported uncertainty from both outcomes.

Collectively, the results satisfy the principal objectives of characterizing the effects of flow and pulsation variables, quantifying heat-transfer behavior, developing validated computational models, deriving dimensionless correlations, and identifying a statistically robust operating condition. The variables of velocity, Reynolds, Prandtl, Strouhal, Fourier, frequency, amplitude and duty cycle aid in fluid-flow characterization. The heat transfer characterization is facilitated using the Nusselt number and enhancement ratio. The computational goals are achieved by using strong factorial modeling, blocked validation, dimensionless correlation development, a bootstrap analysis, and uncertainty-aware optimization.

The current evidence does not provide a complete thermo-hydraulic optimum since there is no evidence for pressure drop, friction factor, and pumping power in the source data set. Likewise, no CFD temperature or velocity fields were generated. Accordingly, the identified setting represents a validated thermal optimum among the 18 measured combinations rather than a universal optimum outside the investigated in-line tube-bundle domain.

5. DISCUSSION

Results demonstrate that reciprocating pulsation substantially improved heat transfer within the investigated in-line tube bundle. Mean Nusselt number was 58.87, while the mean enhancement ratio was 1.247. Enhancement above steady flow occurred in 95.56% of the 180 regimes, and 81.67% achieved an improvement of at least 10%. Pulsation was therefore generally beneficial, although five regimes produced $Nu_p/Nu_{st} < 1$, confirming that enhancement depends on the selected operating condition.

Frequency produced a strong positive response within the measured range. Mean Nu increased from 52.63 at 0.180 Hz to 64.89 at 0.450 Hz, while mean enhancement increased from 1.113 to 1.377. Higher frequency likely accelerated boundary-layer renewal and increased mixing between wake and core-flow regions. Huang and Jiang similarly reported that oscillating-flow heat transfer depends on the coupled influence of flow intensity, oscillation timescale, fluid properties, and geometry [22]. Amplitude ratio produced a comparable effect. Increasing A/D from 5 to 15 raised mean Nu from 53.70 to 63.77 and enhancement from 1.135 to 1.353. Greater displacement probably intensified gap-flow acceleration, wake interaction, and thermal mixing. Comparable changes in wake behaviour and heat transfer under pulsating flow have been reported around heated cylinders [23].

A duty cycle of 0.2 produced higher mean Nu and enhancement than a duty cycle of 0.5. Shorter active periods may have imposed sharper accelerations and stronger transient shear. Factorial ANCOVA confirmed significant frequency, amplitude, and duty-cycle effects after controlling for Reynolds and Prandtl numbers. Reynolds number dominated absolute Nu, whereas its association with relative enhancement was negative. Pulsation therefore produced its greatest proportional benefit where baseline steady-flow convection was comparatively lower. Significant frequency-amplitude interactions further showed that these factors should be optimised jointly.

Blocked validation provided strong evidence of model transferability within the measured domain. Nusselt prediction achieved $R^2 = 0.9779$ and MAPE of 2.56%, while enhancement prediction achieved $R^2 = 0.7782$ and MAPE of 4.76%. Strouhal-based dimensionless correlations also performed well, with blocked R^2 values of 0.9593 for Nu and 0.8431 for enhancement. Such

performance supports their use for preliminary prediction within the tested ranges. Thermal management modelling of batteries has also been shown to be useful from a computational perspective for developing desirable cooling conditions using pulsation-based models [24].

Optimal measured setting was 0.450 Hz, $A/D = 15$, and $\psi = 0.2$. It generated $Nu = 71.63$ and $Nu_p/Nu_{st} = 1.520$, which are improvements of about 52.7% compared to the reference with low pulsation. Decision stability was high for the ten held-out blocks and 189 sensitivity scenarios.

There are significant limitations of engineering interpretation. Only one geometry of the tube bundle, one working fluid and 18 combinations of pulsations were investigated. There was no information on pressure drop, friction factor, pumping power, local temperature fields or velocity distributions. Thus, the chosen regime is only a thermal optimum and not a thermo-hydraulic optimum. Studies of pulsating flow have demonstrated that better heat transfer can be realized with increased pressure drop [25]. The results should be expanded to other measured domains in the future, and extended to other hydraulic domains, bundle arrangements, time-resolved experiments, and validated transient CFD.

5. CONCLUSION

A combined experimental and computational study was performed to study the fluid-flow and heat transfer behavior of an in-line tube bundle subjected to reciprocating pulsating cross-flow. The analysis was performed on 180 experimental regimes corresponding to 18 combinations of frequency, amplitude and duty cycle for 10 different Reynolds-Prandtl operating blocks. The use of factorial ANCOVA, HC3 inference, uncertainty weighted regression, blocked validation, dimensionless correlation development, bootstrapping and multi-response optimization were incorporated into a reproducible framework. The heat transfer was improved by the pulsating flow in 95.56% of the measured regimes and at least 10% improvement was obtained in 81.67% of the measured regimes. The mean Nusselt number was 58.87, and mean enhancement ratio was 1.247. The strongest controllable factors were the frequency/amplitude ratio, with duty cycle having a smaller but consistent effect, with $\psi = 0.2$ being better than $\psi = 0.5$. The absolute Nusselt number was mainly influenced by Reynolds and Prandtl numbers whereas the relative enhancement was more sensitive to the pulsation parameters. The R^2 value obtained for Nusselt number and enhancement were 0.9779 and 0.7782 with MAPE value of 2.56% and 4.76% respectively in the case of blocked validation. The blocked R^2 values of the Strouhal based correlations were 0.9593 and 0.8431, respectively, and the results were quite good. The best tested setting was $f = 0.450\text{Hz}$, $A/D = 15$, and $\psi = 0.2$, giving $Nu = 71.63$ and $Nu_p/Nu_{st} = 1.520$ which is about 52.7% more than the low-pulsation reference. It was selected in all the validation folds and sensitivity scenarios, thus demonstrating good thermal optimality. Future research should include pressure drop, pumping power, structural vibration, other geometries, wider range of pulsation, transient CFD and independent experimental validation, to provide full thermo-hydraulic performance and wider applicability.

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