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DOUBLE ZZ-SHEHU TRANSFORM AND APPLICATION IN MATHEMATICAL MODELS

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ABSTRACT
In this work, we presented a double ZZ-Shehu transform and used it as a tool to solve certain mathematical systems that represent mathematical models describing the volumetric blood flow and pressure distribution at the ends of a blood vessel. We used the brachial artery and the coronary arteries as supporting examples.

KEYWORDS: Double ZZ-Shehu transform; Mathematical model; Navier-stokes equation; brachial artery; coronary arteries.

1-INTRODUCTION

Partial differential equations (PDEs) serve as the fundamental cornerstone for the mathematical characterization of intricate phenomena within the fields of medicine, engineering, and biophysics. In this context, contemporary research has increasingly shifted toward adopting advanced computational and numerical frameworks to derive precise estimations of bio fluid dynamics variables—specifically volume, pressure, and flow rates—due to their robust capability in simulating highly dynamic systems.

The technique of developing a mathematical model is called mathematical modeling. The science dedicated to describing the physics of blood flow is called hemodynamics. This study will focus on the mathematical modeling of blood flow problems. Although blood is generally considered a non-Newtonian fluid [1, 2], we will treat it as a Newtonian fluid (where blood properties become linear) and apply the Navier-Stokes equation to solve blood flow problems [3-5].

Given the mathematical hurdles associated with resolving initial and boundary value problems (IBVPs) in such environments, integral transforms—including Laplace, Elzaki, ZZ, and Shehu—have proven their efficacy as analytical instruments for simplifying complex differential equations [6-9]. These transformations have found applications in various sciences and have even extended to financial sciences [10-12]. Recently, double integral transforms have gained significant scholarly attention due to their direct methodologies in achieving solutions [13-15].

In this research, we introduce a double integral transform that synthesizes the properties of the ZZ and Shehu transforms. This hybrid transform is employed to address the integral equations governing hemodynamics, demonstrating exceptional efficiency in calculating flow velocity and pressure gradients, particularly under complex initial conditions or temporal overlaps resulting from the successive phases of the cardiac cycle.

2- PRELIMINARIES

Definition: The double ZZ-Shehu (ZS) of the function £ (r, t) is represented by Ψ_2 and is expressed in the following manner[ ]

$\Psi_2\{(£(\mathrm{r,t})\} = ZS[(\theta,\delta),(\omega,\mu)] = \frac{\theta}{\omega}\int_0^\infty\int_0^\infty e^{-(\frac{\theta}{\omega}r+\frac{\delta}{\mu}t)}$ £(r, t) dr dt

the inverse double ZZ- Shehu transform of Ψ_2 is defined as:

$\Psi_2^{-1}\{\Psi_2\{£(\mathrm{r,t})\}\} = ZS^{-1}\left(ZS[(\theta,\delta),(\omega,\mu)]\right)$

$= \frac{1}{2\pi i}\int_{\epsilon-i\infty}^{\epsilon+i\infty}\frac{\theta}{\omega}ZS[(\theta,\delta),(\omega,\mu)]e^{-\frac{\theta}{\omega}r}\,d\omega\ .\frac{1}{2\pi i}\int_{\epsilon-i\infty}^{\epsilon+i\infty}\frac{1}{\mu}$

$ZS[(\theta,\delta),(\omega,\mu)]e^{-\frac{\delta}{\mu}t}\,d\delta$

where ZS^{-1} is the inverse of transform operator.

The linear characters of the double $\Psi_2$transform**.**

$$\Psi_2\,\{a\,£(r,t)+b\beta(r,t)\} = a\,\Psi_2\left(£\,(r,t)\right)+b\,\Psi_2(\beta\,(r,t))$$

From definition

$\Psi_2\,\{a\,(£(r,t)+b\,\beta(r,t)\}$

$$= a\frac{\theta}{\omega}\int_0^\infty\int_0^\infty \{£(r,t)\}\, e^{-\left(\frac{\theta}{\omega}r+\frac{\delta}{\mu}t\right)}\ drdt\ +\ b\frac{\theta}{\omega}\int_0^\infty\int_0^\infty \beta(r,t)\ e^{-\left(\frac{\theta}{\omega}r+\frac{\delta}{\mu}t\right)}\ drdt$$

$$=\ a\Psi_2\left(£\,(r,t)\right)+b\,\Psi_2(\beta\,(r,t))$$

Theorem: The double Ψ_2 transform of first and second order partial derivatives is expressed in the following manner:

(1) $\Psi_2\left\{\dfrac{\partial\, £(r\,,t)}{\partial r}\right\} = \left(\dfrac{\theta}{\omega}\right)\Psi_2\big(£(r\,,t)\big)-(\dfrac{\theta}{\omega})S(£(0,t)$

(2) $\Psi_2\left\{\dfrac{\partial^2\, £(r\,,t)}{\partial^2 r}\right\} = \left(\dfrac{\theta^2}{\omega^2}\right)\Psi_2\big(£\,(r,t)\big)\ -\left(\dfrac{\theta^2}{\omega^2}\right)S\big(£(0\,,t)\big)-\left(\dfrac{\theta}{\omega}\right)\ \left(\dfrac{\partial}{\partial r}\ Z(£\,(0\,,t)\right)$

(3) $\Psi_2\left\{\dfrac{\partial £\,(r\,,t)}{\partial t}\right\} = (\dfrac{\delta}{\mu})\ \Psi_2\big(£(r\,,t)\big)-Z(£(r\,,0))$

(4) $\Psi_2\left\{\dfrac{\partial^2\, £(r\,,t)}{\partial^2 t}\right\} = \left(\dfrac{\delta^2}{\mu^2}\right)\ \Psi_2\big(£(r\,,t)\big)\ -\left(\dfrac{\delta}{\mu}\right)Z\big(£(r\,,0)\big)-Z\ \left(\dfrac{\partial}{\partial t}\ Z(\,r\,,0)\right)$

Proof: The proof is obtained directly using definition of double ZZ- transform [ ].

Table (1): double ZZ-Shehu transformation for some special functions

No	$£(r,t)$	$\Psi_2\{\,£(r,t)\}$
1	1	$\frac{\mu}{\delta}$
2	$e^{(ar+bt)}$	$\frac{\theta\ \ \mu}{(\theta-a\omega)\ \ (\delta-b\mu)}$
3	$r^{\epsilon}\ \ t^{\rho}$	$\epsilon!\ \rho!\left(\frac{\omega}{\theta}\right)^{\epsilon}\ \left(\frac{\delta}{\mu}\right)^{\rho+1}$
4	$r^{\epsilon}\ \ t^{\rho}\quad \epsilon,\rho\geq -1$	$\Gamma(\in+1)\left(\frac{\omega}{\theta}\right)^{\in}\ \ \Gamma(\rho+1)\ \ \left(\frac{\delta}{\mu}\right)^{\rho+1}$
5	$\cos(ar+br)$	$\frac{\theta\mu(\theta\delta+\alpha\mu\omega b)}{(\theta^2+a^2\omega^2)\,(\delta^2+b^2\mu^2)}$
6	$\sin(ar+bt)$	$\frac{\theta\mu(\theta\mu b+\delta\omega a)}{(\theta^2+a^2\omega^2)\,(\delta^2+\mu^2 b^2)}$
7	$\cos h(ar+bt)$	$\frac{\theta\mu(\theta\delta+\alpha\mu\omega b)}{(\theta^2-\alpha^2\omega^2)\,(\delta^2-b^2\mu^2)}$
8	$s\,inh(ar+bt)$	$\frac{\theta\mu(\theta\mu b+a\omega\delta)}{(\theta^2-\alpha^2\omega^2)\,(\delta^2-b^2\mu^2)}$

3- BLOOD FLOW IN VESSELS (POISEUILLE'S LAW)

The following assumptions are used to calculate the volumetric flow of a blood in vessels:

- Newtonian fluid
- Rigid vessel walls
- Steady flow

Since real blood is non-Newtonian and pulsatile, the law serves as an ideal approximation for vessel hemodynamics. When studying blood flow, the blood vessel is sometimes approximated as two parallel, level channels (Plane Channel approximation).

If we have a cylinder of fluid with radius r and length L moving inside a capillary tube with radius a, due to the pressure difference (P_1-P_2)between its two ends, the driving force of the flow cylinder is equal to the viscous force resisting the movement of the vessel, because the flow is constant and uniform

$$\pi r^2(P_1-P_2)=-\mu(2\pi rL)\frac{dv}{dr}$$

apply double ZZ-Shehu transform for the above equation and using boundary condition of the tangential component of the fluid velocity in the film region v = 0 when r = a and $V=v$ at a = 0

$$\Psi_2\{\pi r^2(P_1-P_2)\}=\Psi_2\left\{-\mu(2\pi rL)\frac{dv}{dr}\right\}$$

$$\frac{\mu}{\delta}(P_1-P_2)=\ \beta\left(\frac{\theta}{\omega}\right)\Psi_2\big(V(r\,,t)\big)-\left(\frac{\theta}{\omega}\right)S\big(V(0,t)\big):\ \beta=\ -\frac{2\mu L}{r}$$

$$V(r,t)=\frac{(P_1-P_2)}{4\mu L}a^2 \qquad (1)$$

Since this velocity gradually decreases until it reaches zero at the tube wall, we can consider the fluid to be flowing at an average velocity, with the values:

$$\bar{V} = \frac{V+0}{2} = \frac{1}{2}V$$

where V is the highest value of the velocity, and $\bar{V}$ is the average velocity of the fluid. Since the rate of fluid flow inside a tube, according to (Poiseuille's Law) is $Q = A\bar{V}$, where ($A = \pi a^2$) is cross-sectional area of tube. Substituting V from equation (1), the flow rate Q can be calculated as follows:

$$Q = \frac{(P_1 - P_2)\pi a^4}{8\mu L} = \frac{(P_1 - P_2)}{R} \; ; \; R = \frac{8\mu L}{\pi a^4} \;\; \rightarrow \;\; (P_1 - P_2) = \frac{4\mu L V}{a^2} \qquad (2)$$

R represents the resistance to fluid flow.

To Find the difference in blood pressure as it passes through a capillary of length (2 mm) and diameter (4 μm), given that the blood flow velocity at its axis is (0.57 mm/s) and the viscosity coefficient of blood is $\mu = 4.10^{-3}\text{Ns}/m^2$. Using equation (2)

$$(P_1 - P_2) = \frac{4\times 4.10^{-3}\text{Ns}/m^2 \times 0.57.10^{-3} \times 2.10^{-3}}{(2.10^{-6}m)^2} = \frac{18.24\times 10^{-19}}{4\times 10^{-12}}$$

$$= 4.56 \times 10^3 N/m^2$$

Moreover, to calculate the rate of blood flow in the capillary:

$$R = \frac{8\times 4.10^{-3}\text{Ns}/m^2 \times 2.10^{-3}}{\pi(2.10^{-6}m)^4} = 1.274 \times 10^{18} Ns/m^5$$

$$Q = \frac{4.56\times 10^3 N/m^2}{1.274\times 10^{18} Ns/m^5} = 3.58\times 10^{-15} m^3/s$$

Table (2): This table shows the pressure difference, resistance, and blood flow rate

	Length L	Radiuses a	Pressure difference $(P_1 - P_2)$	Resistance R	Blood flow rate Q
1	2×10^{-3}	2×10^{-6}	4.56×10^3	1.27×10^{18}	3.58×10^{-15}
2	4×10^{-3}	1.9×10^{-6}	10.11×10^3	3.2×10^{18}	3.16×10^{-15}
3	6×10^{-3}	1.8×10^{-6}	16.89×10^3	5.83×10^{18}	2.89×10^{-15}
4	8×10^{-3}	1.7×10^{-6}	25.25×10^3	9.76×10^{18}	2.58×10^{-15}
5	10×10^{-3}	1.6×10^{-6}	35.6×10^3	15.55×10^{18}	2.29×10^{-15}
6	12×10^{-3}	1.5×10^{-6}	48.64×10^3	24.17×10^{18}	2.012×10^{-15}
7	14×10^{-3}	1.4×10^{-6}	65.11×10^3	37.15×10^{18}	1.75×10^{-15}
8	16×10^{-3}	1.3×10^{-6}	86.34×10^3	57.09×10^{18}	1.51×10^{-15}
9	18×10^{-3}	1.2×10^{-6}	113.9×10^3	88.46×10^{18}	1.286×10^{-15}

The table shows that the pressure difference (ΔP) increases as the vessel length increases, indicating a direct relationship between them. In contrast, the blood flow rate (Q) decreases with increasing length and decreasing radius due to the rise in hydrodynamic resistance. The results also confirm that the radius has a strong influence on flow rate, as even a slight reduction in radius leads to a significant decrease in flow, consistent with the theoretical basis of Poiseuille's law.

4-THE NAVIER–STOKES EQUATION.

Navier stockes equation for fluid flow along an axis corresponds to the properties of blood flow in vessels.

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial Ł}{\partial y} + w\frac{\partial u}{\partial z}\right) = -\frac{\partial P}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)$$

When the velocity components in the x, y, and z directions are u, v, and w, respectively. Where ρ is blood density, P is blood pressure, and μ is the kinematic viscosity of blood. After simplification Naviar stokes equation regarding assumptions of blood flow:

$$\frac{\partial u}{\partial t} = -\frac{\partial P}{\partial x} + \mu\,\frac{\partial^2 u}{\partial y^2}$$

In [9] authors derived blood flow rate with more details (the reader can return it) as

$$Q = \iint w\partial\gamma = \frac{1}{2} q\pi R^2 \qquad (3)$$

Where w be the axial flow component in z-direction, R is the radius of the blood vessels. Therefore, from (3) with (2) two differential equations are obtained

$$\frac{\partial Q}{\partial t} + \frac{3QL}{V}\frac{\partial Q}{\partial z} - \frac{2LQ^2}{V^2}\frac{\partial V}{\partial z} + \frac{4\pi\mu L}{V}Q + \frac{V}{2\rho L}\frac{\partial P}{\partial z} = 0 \qquad (4)$$

$$\frac{\partial V}{\partial t} + L\frac{\partial Q}{\partial z} = 0 \qquad (5)$$

From the combination of relations (4) and (5), one obtains the differential equation below:

$$\frac{\partial Q}{\partial t} + \frac{3Q}{V}\frac{\partial V}{\partial t} - \frac{2LQ^2}{V^2}\frac{\partial V}{\partial z} + \frac{4\pi\mu L}{V}Q + \frac{V}{2\rho L}\frac{\partial P}{\partial z} = 0 \qquad (6)$$

Assuming V = SL, with S representing the blood vessel cross-sectional area, equation (6) is ansformed into equation (7).

$$\frac{\partial Q}{\partial t}-\frac{3Q}{S}\frac{\partial S}{\partial t}-\frac{2LQ^2}{S^2}\frac{\partial S}{\partial z}+\frac{4\pi\mu}{S}Q+\frac{S}{2\rho}\frac{\partial P}{\partial z}=0 \qquad (7)$$

model of heart dynamics, blood flow, and pressure can be built using logical physical assumptions and validated parameters from previous studies.

The difference in pressure, ∂P/∂z, is 100 to 40 mmHg. Initial value of Q = 1 to 5.4 liter/minute, Kinematic viscosity of blood, μ = 0.035 cm^2/s. Density of blood, ρ =1.043 to 1.057 g/cm^3 [9]

The results indicate that increasing vessel length, together with a gradual reduction in radius, leads to higher resistance and pressure difference, while the flow rate decreases. This behavior confirms the nonlinear relationship between vessel length and the associated hemodynamic variables

5- MATHEMATICAL SOLUTIONS

In the model, the heart is treated as a point element for pumping blood, assuming constant flow and pressure gradient, which simplifies equation (6) to equation (8).

$$\frac{\partial V}{\partial t}-\frac{V^2}{6\rho LQ}\frac{\partial P}{\partial z}-\frac{4\pi\mu L}{3}=0 \qquad (8)$$

Where V is volumetric flow rate. Blood flow is the blood volume per unit time, with constant vessel area and pressure gradient; under these assumptions, equation (7) becomes the flow model in equation (9).

$$\frac{\partial Q}{\partial t}+\frac{4\pi m}{S}Q+\frac{S}{2\rho}\frac{\partial P}{\partial z}=0 \qquad (9)$$

Where Q is blood flow rate, to find the formula of Q can use (Ψ_2) with initial condition $Q(r,0)=Q_0$

$$(\frac{\delta}{\mu})\,\Psi_2\big(Q(r,t)\big)-Z(Q(r\,,0)+\frac{4m}{R^2}\Psi_2 Z(Q(r,t)+\frac{\pi R^2}{2\rho}\frac{\partial P}{\partial z}\frac{\mu}{s}=0$$

$$(\frac{\delta}{\mu}+\frac{4m\mu}{R^2})\Psi_2\big(Q(r,t)\big)=\ Q_0+\frac{\pi R^2}{2\rho}\frac{\partial P}{\partial z}\frac{\mu}{s}$$

$$(\frac{\delta R^2+4m\mu}{\mu R^2})\Psi_2\big(Q(r,t)\big)=Q_0\ +\alpha_1\ ; \qquad \alpha_1=\frac{\pi R^2}{2\rho}\frac{\partial P}{\partial z}\frac{\mu}{s}$$

$$\Psi_2\big(Q(r,t)\big)=(\frac{\mu R^2}{\delta R^2+4m\mu})(Q_0\ +\alpha_1\frac{\mu}{\delta})$$

Thereby, inverse of ZZ-Shehu transform

$$Q(r,t)=Q_1+(Q_0-Q_1)e^{\frac{-t}{\alpha_2}}, \qquad (10)$$

where $\alpha_2=\frac{R^2}{8v};\, v=\frac{\mu}{P}$ $\qquad Q_1=\frac{-\pi R^4}{8\mu}\frac{\partial\rho}{\partial z}$

Blood pressure is the force on the vessel walls during blood circulation, and the Poiseuille model is used to relate the flow rate to the pressure, as shown in equation (11).

$$Q=\frac{\pi R^4}{8L\mu}P \qquad (11)$$

After inserting (11) into (10), the new equations are obtained, and (12) represents the model of blood pressure

$$\frac{\partial P}{\partial t}+\frac{4\mu}{R^2}P+\frac{4L\mu}{\rho R^2}\frac{\partial P}{\partial z}=0 \qquad (12)$$

Using ZZ-Shehu transform with initial condition $P(r,0)=P_0$

$$(\frac{\delta}{\mu}+\frac{4\mu}{R^2})\Psi_2\big(P(r,t)\big)=P_0\ +\frac{4L\mu}{\rho R^2}\frac{\partial\rho}{\partial z}\frac{\mu}{s}$$

$$(\frac{\delta R^2+4\mu^2}{\mu R^2})\Psi_2\big(P(r,t)\big)=P_0\ +\beta_1\ ; \qquad \beta_1=\frac{4L\mu}{\rho R^2}\frac{\partial\rho}{\partial z}\frac{\mu}{s}$$

$$\Psi_2\big(P(r,t)\big)=(\frac{\mu R^2}{\delta R^2+4\mu^2})(P_0\ +\quad {}_1)$$

After taking inverse of Ψ_2

$$P(r,t)=P_0(\mathrm{r}-\beta_3\mathrm{t})e^{-\beta_2 t} \qquad (13)$$

Where $\beta_2=\frac{4\mu}{R^2}$, $\beta_3=\frac{4L\mu}{\rho R^2}$

6- APPLYING MODELS TO A HEALTHY HUMAN.

In this section we will discuss some applications related to the human body.

6-1 Velocity and pressure in the brachial artery

The previously derived physical laws were utilized to apply the model to a healthy, non-smoking 40-year-old male with a pulse rate of 62 beats per minute and a vascular wall thickness of 0.1 mm.

The measurements were obtained using Doppler ultrasound imaging. Using the equations (8) and (13), and calculations the velocity variation with radius variation at every 5cm of the brachial artery length, the average speed is 53.96, as shown in the Table (4).

Table (3): Blood flow dynamics along the brachial artery

Length of the brachial artery L cm	Radius r mm	velocity V cm/sec	Area S mm^2	the pressure P mmHg
0	1.30	51.69	5.3	80
5	1.29	52.69	5.2	79.62
10	1.28	53.7	5.1	79.2
15	1.27	54.8	5.0	78.78
20	1.26	55	4.98	78.78
25	1.25	55.91	4.9	78.1
average		53.96	5.1	

The data indicate that as the length of the brachial artery increases, there is a gradual decrease in radius and cross-sectional area, accompanied by an increase in blood velocity and a slight decrease in pressure. These findings reflect the natural distribution of velocity and pressure according to the principles of flow continuity and the hemodynamic laws governing blood vessels.

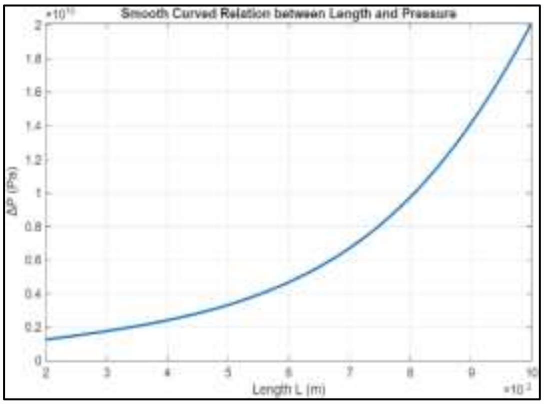

Figure (1): A curve illustrating the direct relationship between pressure and artery length.

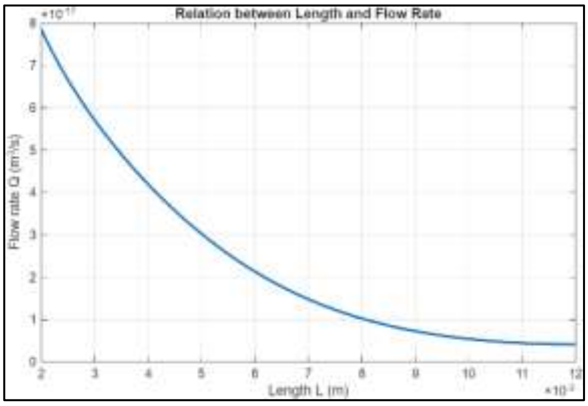

Figure (2): A curve showing the decreasing flow rate as the length decreases

6.2- Pressure and blood flow rate on cardiac arteries

In this instance, the focus is on the left anterior descending artery, the right coronary artery, and the left circumflex artery, which represent critical areas for blood pressure, ensuring a regular and efficient supply of oxygen-rich blood to the heart muscle. Applying equations (10) and (13) to calculate the pressure and flow of the arteries under study.

Table (4): Hemodynamic Parameters and Blood Flow Characteristics of Main Coronary Arteries

Artery	Radius r	Length L	blood flow rate Q	Pressure P
LAD	0.15	3	95.86	43.4
RCA	0.13	2.5	115.14	77.0
LCX	0.11	3.5	82.24	150.2

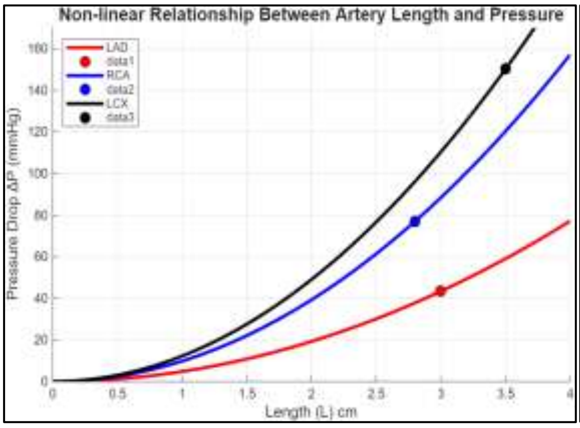

Figure (3): Direct correlation between artery length and hydraulic pressure drop.

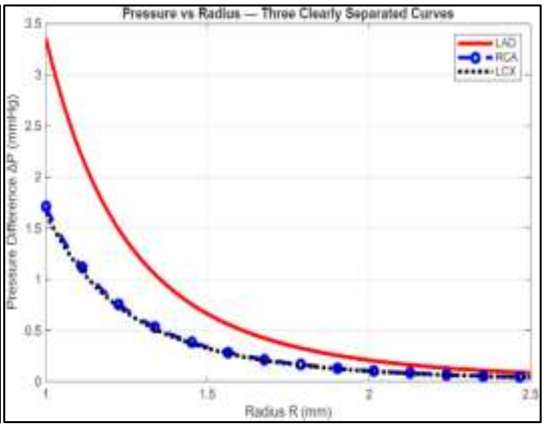

Figure (4): Inverse Relationship Between Artery Radius and Pressure Drop

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