The Original

GLOBAL FAIR DOMINATION NUMBER OF SOME GRAPHS

A. Abithabala[1] , K. Karuppasamy[2]

[1,]Research Scholar, Kalasalingam Academy of Research and Education, Anand Nagar, Krishnankoil-626 126, INDIA,
e-mail:abithabala2001@gmail.com, ORCID ID- 0009-0001-6043-9205
[2] Professor,Kalasalingam Academy of Research and Education, Anand Nagar, Krishnankoil-626 126., INDIA
e-mail:, karuppasamyk@gmail.com, ORCID ID- 0000-0002-7119-0599

Abstract
Let G=(V,E) be a finite, simple, and undirected graph. In this paper, we present a new concept in graph theory known as the global fair domination number. A fair dominating set S in a graph G=(V,E) is called a global fair dominating set if it also functions as a fair dominating set in the complement graph $\overline{G}$. The smallest cardinality of such a set is referred to as the global fair domination number of G, denoted by $\gamma_{gfd}(G)$. In this study, we investigate global fair dominating sets and determine the global fair domination number for various classes of graphs, including path graphs, cycle graphs, wheel graphs, star graphs, ladder graphs, k-partite graphs, and prism graphs. The obtained results reveal recurring structural patterns that facilitate both efficient and balanced coverage in graph-based systems. From an application perspective, these results provide a mathematical framework for modeling medical networks, where healthcare facilities and their communication or referral links can be represented as vertices and edges, respectively. The fairness condition promotes equitable healthcare service coverage, while the global requirement enhances the robustness of the medical network under both normal and alternative connectivity conditions.

KEYWORDS: Domination, global domination, fair domination, fair global domination, path graph, cycle graph, wheel graph, Medical Network.

1. INTRODUCTION

Domination theory is an important part of graph theory because it has many practical applications, such as designing networks, allocating resources, and building monitoring systems. A comprehensive exploration of domination concepts and their various forms can be found in the foundational work by Haynes, Hedetniemi, and Slater [3]. As time has passed, many new types of domination have been developed to address real-world problems in network environments. One important type is fair domination, introduced by Caro, Hansberg, and Henning [1]. This version adds a fairness rule to make sure domination is spread out fairly among all parts of the network. This idea has been studied in different types of graphs and with different parameters, such as fair bondage numbers and related structures [4, 5]. These studies show the value of making sure domination is balanced, not just covering everything. Another key extension is global domination, which requires a dominating set to work well in both a graph and its opposite. This idea was introduced by Sam pathkumar [6] and makes the usual domination idea stronger by ensuring that the network remains functional even if the structure changes. We look into global fair domination, which merges fairness with full coverage into one idea.

This idea is built on two key ideas: fairness and complete coverage. In this approach ,a dominating set needs to cover Both the graph and its opposite, while keeping the coverage balanced.This idea especially helpful in situations where fair sharing of resources and dependable service are important.Domination models are used a lot in real-life situations, such as healthcare systems, distributing medical resources ,and planning emergency services.Looking at different types of graphs, like split graphs and path graphs, helps us understand how global fair domination works structurally.This idea is especially useful in medical networks, where hospitals, clinics, or healthcare centers are treated as points, and connections like communication or referrals are the links between them.In this case, a global fair dominating set helps identify healthcare places that offer fair service coverage and keep things running smoothly even when the network changes.The fairness part helps in sharing medical resources evenly, and the global part makes the medical network more reliable, which is important for emergency referrals, telemedicine, and planning healthcare systems. In this work, we examine the global fair domination number and find its precise values for basic kinds of graphs, including cycle graphs, path graphs, and certain graphs [2]. Additionally, we analyze the consequences of these structural patterns in network-based applications and establish them for the relevant dominating sets.

Definition 1.1. [1] A Fair dominating set in a graph G (or FD set) is a dominating set S such that all vertices not in S are dominated by the same number of vertices from S; that is every two vertices not in S have the same number of neighbours in S. The fair domination number$\mathrm{fd}(G)$, of G is the minimum cardinality of all FD set.

Definition 1.2. [6] A dominating set of a graph $G = (V, E)$ is a Global dominating set if S is also dominating set of the complement $\overline{G}$ of G. The global domination number $\gamma_g(G)$of G is the minimum cardinality of a global dominating set. In the next part, we will discuss the concept of global fair domination and calculate the global fair domination number $\gamma_{gfd}(G)$ for some basic graphs such as path, cycle, wheel, star, Petersen graph, complete k-partite graph, ladder, and prism graphs.

2. Main Results

Definition 2.1. Let $G = (V, E)$ be a simple graph and let $\overline{G}$ denote the complement of G. A set $S \subseteq V(G)$ is called a global fair dominating set (or GFD-set) if S is a fair dominating set of both G and its complement graph that is, for any two vertices $u, v \in V(G)S$,

$$|N_G(u) \cap S| = |N_G(v) \cap S|$$
$$|N_{\overline{G}}(u) \cap S| = |N_{\overline{G}}(v) \cap S|$$

The minimum cardinality of a global fair dominating set of G is called the global fair domination number of G and is denoted by$\gamma_{gfd}(G)$.We proceed to determine the global fair domination number of some standard graphs. The following observation is useful in this regard.

Observation 2.2. Let G be a graph of order n. Then the following hold.

(a) $\gamma_{gfd}(G) \geq \max\{\gamma(G), \gamma(\overline{G})\}$
(b) $\gamma_g(G) \leq \gamma_{gfd}(G) \leq n$.
(c) $\gamma_{gfd}(G) = n$ if and only if $G = K_n$.
(d) If S is a fair global dominating set of G, then S is a dominating set of both G and$\overline{G}$.
(e) Every fair global dominating set of G is a fair dominating set of G.

Theorem 2.3. Let P_n be a path graph on n vertices, $n \geq 3$. Then the global fair domination number of P_n is

$$\gamma_{gfd}(P_n) = \begin{cases} 2, n = 3, \\ \left\lceil \frac{n}{3} \right\rceil, n \geq 4. \end{cases}$$

Proof. Let $P_n = (v_1, v_2, \ldots, v_n)$ be a path graph with the dominating set S. For n = 3, at least two vertices are required to dominate both P_3 and its complement $\overline{P_3}$ and also satisfying the fairness condition. Hence $\gamma_{gfd}(P_3) = 2$.Now consider P_n, $n \geq 4$. There are three cases in S. Hence S is of any one of the forms.

$$S = \begin{cases} \bigcup_{k=0}^{\frac{n}{3}-1} \{v_{3k+2}\} & \text{for} n \equiv 0(\text{mod}3) \\ \bigcup_{k=0}^{\left\lfloor \frac{n}{3} \right\rfloor-1} \{v_{3k+2}\} \cup \{v_{n-1}\} & \text{for} n \equiv 1(\text{mod}3) \\ \bigcup_{k=0}^{\left\lfloor \frac{n}{3} \right\rfloor-1} \{v_{3k+2}\} \cup \{v_n\} & \text{for} n \equiv 2(\text{mod}3) \end{cases}$$

Case 1: If $n \equiv 0 \pmod 3$. Let n = 3k. The path P_n can be partitioned into k blocks of three consecutive vertices. Let S = {v2, v5, v8, . . . }, the set of middle vertices from each block dominates all vertices of Pn and its complement graph $\overline{P_n}$ and satisfying the fairness condition and also it is minimum. Thus, we get $\gamma_{gfd}(P_n) = k = \frac{n}{3}$.

Case 2: If $n \equiv 1 \pmod 3$. Let n = 3k + 1. The path P_n can be partitioned into k blocks of three consecutive vertices and with one additional vertex. Let $S = \{v_2, v_5, v_8, \ldots\} \cup \{v_{n-1}\}$ dominates P_n and its complement $\overline{P_n}$ and satisfying the fairness condition and also it is minimum. Thus, we get
$\gamma_{gfd}(P_n) = k + 1 = \frac{n+2}{3}$.

Case 3: If $n \equiv 2 \pmod 3$. Let n = 3k+2. The path P_n can be partitioned into k blocks of three consecutive vertices and extra two vertices. Let $S = \{v_2, v_5, v_8, \ldots\} \cup \{v_n\}$ dominates all vertices of P_n and its complement $\overline{P_n}$, and satisfying the fairness condition and also it is minimum. Hence

$$\gamma_{gfd}(P_n) = k + 1 = \frac{n+1}{3}$$

Hence we get $\gamma_{gfd}(P_n) = \left\lceil \frac{n}{3} \right\rceil, n \geq 4$.

Theorem 2.4. Let C_n be a cycle graph on n vertices, where $n \geq 4$. Then

$$\gamma_{gfd}(C_n) = \begin{cases} \frac{n}{3}, n \equiv 0(\text{mod}3), \\ \frac{n+2}{3}, n \equiv 1(\text{mod}3), \\ \frac{n+4}{3}, n \equiv 2(\text{mod}3). \end{cases}$$

Proof. Let $V(C_n) = \{v_1, v_2, \ldots, v_n\}$.Now, we consider C_n, hence S is of any one of the form

$$S = \begin{cases} \bigcup_{k=0}^{\frac{n}{3}-1} \{v_{3k+2}\} & \text{for} n \equiv 0 (\text{mod} 3) \\ \bigcup_{k=0}^{\lfloor \frac{n}{3} \rfloor -1} \{v_{3k+2}\} \cup \{v_{n-1}\} & \text{for} n \equiv 1 (\text{mod} 3) \\ \bigcup_{k=0}^{\lfloor \frac{n}{3} \rfloor -1} \{v_{3k+2}\} \cup \{v_{n-2}, v_{n-1}\} & \text{for} n \equiv 2 (\text{mod} 3) \end{cases}$$

Case 1: If $n \equiv 0$ (mod 3). Let $n = 3k$. The cycle C_n blocks of three consecutive vertices. Choose the middle vertex from each block $S = \{v_2, v_5, v_8, \dots, v_{3k-1}\}$ The selected vertices The The selected vertices dominates the cycle graph C_n and its complement $\overline{C_n}$ and satisfying the fairness condition and also it is minimum. Hence, $\gamma_{gfd}(C_n) = k = \frac{n}{3}$.

Case 2: If $n \equiv 1$ (mod 3). Let $n = 3k + 1$. The cycle C_n can be partitioned into k blocks of three consecutive vertices with one additional vertex.

Let $S = \{v_2, v_5, v_8, \dots\} \cup \{v_{n-1}\}$dominates C_n and $\overline{C_n}$ and satisfying the fairness condition and also it is minimum. Thus, we get

$\gamma_{gfd}(C_n) = k + 1 = \frac{n+2}{3}$.

Case 3: If $n \equiv 2$ (mod 3). Let $n = 3k + 2$. The cycle C_n can be partitioned into k blocks of three consecutive vertices with two additional vertex. Let $S = \{v_2, v_5, v_8, \dots\} \cup \{v_{n-2}, v_{n-1}\}$ dominates C_n and$\overline{C_n}$and also satisfying the fairness condition and also it is minimum. Hence, we get

$\gamma_{gfd}(C_n) = k + 2 = \frac{n+4}{3}$ Hence the result.

Theorem 2.5. Let W_n be a wheel graph on $n \geq 5$ vertices. Then the global fair domination number of W_n is

$$\gamma_{gfd}(W_n) = \begin{cases} 2\lfloor \frac{n}{6} \rfloor + 2, & \text{if} n \text{is even}, \\ 2\lfloor \frac{n-3}{6} \rfloor + 3, & \text{if} n \text{is odd}. \end{cases}$$

Proof. Let W_n consist of a rim$V(W_n) = \{v_0, v_1, v_2, \dots, v_{n-1}\}$ and v_0 is a central vertex.

Case 1: when n is even In this case, the wheel can be partitioned the rim into blocks of three consecutive vertices and with one extra vertex. choose the vertex set

$$S = \bigcup_{k=0}^{\lfloor \frac{n-1}{3} \rfloor -1} \{v_{3k+2}\} \cup \{v_0, v_{n-2}\}$$

dominates and satisfying the fairness condition in W_n and its complement $\overline{W_n}$ and the set is minimum. Thus, we get $\gamma_{gfd}(W_n) = 2\left\lfloor \frac{n}{6} \right\rfloor + 2$.

Case 2: when n is odd In this case, the wheel can be partitioned the rim into blocks of three consecutive vertices and with two extra vertices. choose the vertex set

$$S = \bigcup_{k=0}^{\lfloor \frac{n-1}{3} \rfloor -1} \{v_{3k+2}\} \cup \{v_0, v_{n-2}, v_{n-3}\}$$

dominates and satisfying the fairness condition in W_n and its complement $\overline{W_n}$ and the set is minimum. Thus, we get $\gamma_{gfd}(W_n) = 2\left\lfloor \frac{n-3}{6} \right\rfloor + 3$. Hence the result. □

Note 2.6. For n = 4, the wheel graph W_4 is isomorphic to the complete graph K_4.
Since $\gamma_{gfd}(G) = n \Leftrightarrow G \cong K_n$, we obtain
$\gamma_{gfd}(W_4) = \gamma_{gfd}(K_4) = 4$. Hence, the above theorem is valid for $n \geq 5$.

Theorem 2.7. The global fair domination number of a star graph $K_{1,n}$ is 2.

Proof. Let $K_{1,n}$ be a star graph with central vertex v and pendant vertices $\{v_1, v_2, \dots, v_n\}$. Consider the set $S = \{v, v_1\}$. Every vertex not in S is adjacent only to v, hence each is dominated by exactly one vertex of S. Thus, the fairness condition is satisfied.

In $\overline{K_{1,n}}$ each vertex in the set $V - S$ is adjacent to exactly one vertex in the set S and satisfying the domination and fairness condition. Therefore, S is a global fair dominating set. Since no single vertex satisfies both domination and fairness conditions, this set is minimum. Hence, the global fair domination number of $K_{1,n}$ is 2.

Thorem 2.8. The global fair domination number of the Petersen graph P is 4.

Proof. Let $V(P) = \{u_1, u_2, u_3, u_4, u_5, v_1, v_2, v_3, v_4, v_5\}$ where $\{u_1, \dots, u_5\}$ form the outer cycle and $\{v_1, \dots, v_5\}$ form the inner vertices. Let $S = \{u_2, u_4, v_1, v_5\}$ Every vertex in $V - S$ is dominated by exactly two vertices of S and satisfying fairness condition. In the complement graph $\overline{P}$, Every vertex in $V - S$ is dominated by exactly two vertices of S and satisfying fairness condition. Therefore, S is a global fair dominating set. Since no smaller set less than 4 can satisfy both domination and fairness conditions in P and $\overline{P}$, this set is minimum. Hence, the global fair domination number of P is 4.

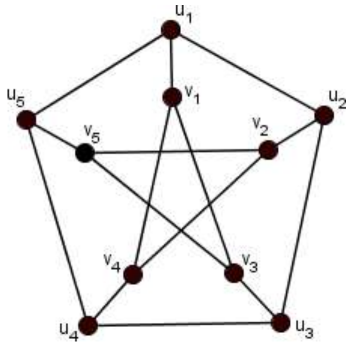

Figure 1: Petersen graph

Theorem 2.9. Let $K_{n_1,n_2,\ldots,n_k}$ be a complete $k-$ partite graph with $n_i \geq 1$ for all $i = 1,2,\ldots,k$. Then $\gamma_{gfd}(K_{n_1,n_2,\ldots,n_k}) = k$.
Proof. Let the vertex set of $K_{n_1,n_2,\ldots,n_k}$ be partitioned as $V = V_1 \cup V_2 \cup \cdots \cup V_k$, where $V_i = \{v_{i1}, v_{i2}, \ldots, v_{in_i}\}, i = 1,2,\ldots,k$. In this graph, every vertex in one partition is adjacent to all the vertices of the other partitions. Consider the set $S = \{v_{11}, v_{21}, \ldots, v_{k1}\}$. Every vertex in $V \setminus S$ is adjacent to exactly $k-1$ vertices in S, thereby satisfying the fairness condition. Moreover, every vertex of S dominates itself and all vertices belonging to the remaining partitions. Hence, S is a dominating set of $K_{n_1,n_2,\ldots,n_k}$. Now consider the complement graph $\overline{K_{n_1,n_2,\ldots,n_k}}$.
In $\overline{K_{n_1,n_2,\ldots,n_k}}$, each partition V_i induces a complete graph, while there are no edges between distinct partitions. Since S contains exactly one representative from each partition, every vertex in $V \setminus S$ is adjacent to exactly one vertex of S in the complement graph, satisfying the fairness condition. Therefore, S is a global fair dominating set. Finally, any set containing fewer than k vertices cannot include a representative from every partition. Hence, at least one partition has no representative in the set, and the vertices of that partition fail to satisfy the global fair domination condition. Thus, no smaller global fair dominating set exists. Hence, $\gamma_{gfd}(K_{n_1,n_2,\ldots,n_k}) = k$.

Theorem 2.10. Let $L_n = P_n \square K_2$ be the ladder graph with $n \geq 2$. Then the global fair domination number of L_n is $\gamma_{gfd}(L_n) = \left\lfloor \frac{n}{2} \right\rfloor + 1$
Proof.
Let $V(L_n) = \{v_1, v_2, \ldots, v_n, u_1, u_2, \ldots, u_n\}$ be the vertex set and
$E(L_n) = \bigcup_{i=1}^{n-1} \{u_i u_{i+1}\} \cup \bigcup_{i=1}^{n-1} \{v_i v_{i+1}\} \cup \bigcup_{i=1}^{n} \{v_i u_i\}$.
be the edge set.
Case 1: when n is even If n is even, Let $S = \{v_1, v_2, v_6, v_{10}, \ldots\} \cup \{u_4, u_8, u_{12}, \ldots\}$. The selected vertices dominates L_n and its complement $\overline{L_n}$ and satisfying the fairness condition and also it is minimum.
$\gamma_{gfd}(L_n) = \left\lfloor \frac{n}{2} \right\rfloor + 1$.
Case 2: when n is odd If n is odd, Let $S = \{v_1, v_5, v_9, \ldots\} \cup \{u_3, u_7, u_{11}, \ldots\}$The selected vertices dominates L_n and its complement $\overline{L_n}$ and satisfying the fairness condition and also it is
$\gamma_{gfd}(L_n) = \left\lfloor \frac{n}{2} \right\rfloor + 1$.
Hence, S is a global fair dominating set. Finally, any set smaller than S will not satisfy the global fair domination condition , this set is minimum. Therefore,
$\gamma_{gfd}(L_n) = \left\lfloor \frac{n}{2} \right\rfloor + 1$.

Theorem 2.11. Let CL_n be the prism graph with $n \geq 4$. Then

$$\gamma_{gfd}(G) = \begin{cases} \frac{n}{2}, n \equiv 0(\mathrm{mod}4), \\ \frac{n+1}{2}, n \equiv 1(\mathrm{mod}4), \\ \frac{n}{2} + 1, n \equiv 2(\mathrm{mod}4), \\ \frac{n+3}{2}, n \equiv 3(\mathrm{mod}4). \end{cases}$$

Proof. Let $V(G) = \{v_1, v_2, \ldots, v_n\} \cup \{u_1, u_2, \ldots, u_n\}$
Case 1: If $n \equiv 0 \pmod 4$. Let $n = 4k$. Choose the set

$S = \bigcup_{i=0}^{k-1} \{v_{4i+1}, u_{4i+3}\}$. vertices, each vertex in $V - S$ dominates the vertices in the set S and satisfy the fairness condition

in both G and its complement graph $\bar{G}$ and this set is minimum.
Thus, we get $|S| = 2k = \frac{n}{2}$.
Case 2 : If n ≡ 0 (mod 4). Let n = 4k+1. Choose the set $S = \bigcup_{i=0}^{k-1}\{v_{4i+3}, u_{4i+3}\} \cup \{v_n\}$
Vertices, each vertex in V-S dominates the vertices in the set S and satisfy the fairness condition in both G and its complement graph $\bar{G}$ and this set is minimum.Thus,
$|S| = 2k + 1 = \frac{n+1}{2}$.
Case 3: If $n \equiv 2 \pmod 4$. Let $n = 4k + 2$. Choosing vertices in the pattern
$S = \bigcup_{i\geq 1}\{v_{4i+3}, u_{4i+1}\} \cup \{v_1, v_2, v_3\}$
vertices, each vertex in $V - S$ dominates the vertices in S and satisfy the fairness condition in both G and its complement $\bar{G}$ and this set is minimum.Thus, we get $|S| = 2k + 2 = \frac{n}{2} + 1$.
Case 4: If $n \equiv 3 \pmod 4$. Let $n = 4k + 3$. Choosing vertices in the pattern
$S = \bigcup_{i\geq 1}\{v_{4i+4}, u_{4i+2}\} \cup \{v_1, v_2, v_3, v_4\}$
The selected vertices dominates and satisfying the fairness condition in G and its complement $\bar{G}$ and this set is minimum. Thus, S is a global fair dominating set. Hence, we get
$|S| = 2k + 3 = \frac{n+3}{2}$. Hence the result.

Note 2.12. For the ladder graph$(P_n \square P_2)$ and the prism graph $(C_n \square P_2)$, the following inequalities hold:
$\gamma_{gfd}(P_n \square P_2) \leq \gamma_{gfd}(P_n)\gamma_{gfd}(P_2)$, and
$\gamma_{gfd}(C_n \square P_2) \leq \gamma_{gfd}(C_n)\gamma_{gfd}(P_2)$.

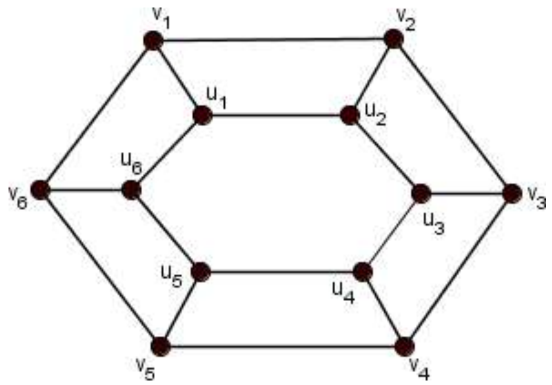

Figure 2: prism graph (CL_6)

3.Application of Global Fair Domination in Healthcare Networks
Healthcare systems can be naturally modeled as graphs, where each vertex represents a healthcare facility such as a hospital, clinic, diagnostic center, or primary healthcare unit. The edges represent transportation routes, communication links, patient referral pathways, ambulance networks, or telemedicine connections between these facilities. Such graph-based representations provide a mathematical framework for analyzing the efficiency, accessibility, and resilience of healthcare service delivery. Within this framework, a dominating set corresponds to a selected group of major healthcare facilities such that every other facility is directly connected to at least one selected center. This ensures complete medical coverage while minimizing the number of service hubs required, thereby reducing operational costs and improving resource utilisation.
A fair dominating set introduces an additional equity constraint by requiring every non-selected healthcare facility to receive the same level of service from the selected centers. This balanced allocation promotes equitable access to healthcare resources, reduces disparities between urban and rural regions, and prevents excessive dependence on a small number of medical institutions.
A global dominating set further enhances the robustness of the healthcare network by ensuring that domination is preserved in both the original network and its complement. In practical terms, this guarantees continuous healthcare coverage not only during normal operating conditions but also when portions of the network become unavailable due to natural disasters, road closures, communication failures, equipment breakdowns, or other emergency situations. The concept of a global fair dominating set combines the advantages of fairness and robustness. It identifies an optimal collection of healthcare facilities that simultaneously provides equitable medical support to all remaining facilities while maintaining network-wide coverage under both normal and disrupted conditions. Consequently, this approach offers a mathematically rigorous strategy for designing resilient and balanced healthcare infrastructures. The proposed model has numerous practical applications in modern healthcare systems, including:
- Designing efficient emergency referral and trauma-care networks.
- Determining optimal locations for ambulance stations and emergency response centers.

- Planning the allocation of intensive care units (ICUs), ventilators, and other critical medical resources during pandemics.
- Selecting vaccination centers to maximize population coverage.
- Organizing telemedicine and digital healthcare service networks.
- Optimizing patient transfer systems between hospitals.
- Developing disaster-resilient healthcare infrastructures capable of maintaining uninterrupted medical services.

Therefore, the theory of global fair domination provides a powerful mathematical tool for optimizing healthcare network design. By simultaneously ensuring equitable resource distribution, minimizing infrastructure costs, and maintaining service continuity under adverse conditions, it contributes significantly to improving the efficiency, accessibility, and resilience of modern healthcare systems.

Conclusion and Scope. In this study, we determined the global fair domination number for several classes of graphs and investigated the structural properties of global fair dominating sets. The obtained results establish exact values and provide a better understanding of the behavior of global fair domination in different graph classes. The concept of global fair domination offers a balance between fairness and robustness, making it a useful mathematical framework for modeling medical networks, where vertices represent healthcare facilities and edges denote communication or referral links. This model has potential applications in equitable healthcare service coverage, emergency referral planning, medical resource allocation, and the design of resilient healthcare infrastructures that remain effective under both normal and alternative network conditions. Future research may focus on determining the global fair domination number for other graph families, such as trees, grids, and product graphs, investigating its relationship with other domination parameters, and developing efficient algorithms for large and complex graphs. These directions will further strengthen both the theoretical significance and the practical applicability of global fair domination in medical and other real-world networks.

Acknowledgment

The authors sincerely thank the anonymous reviewers for their valuable comments and constructive feedback throughout the review process. Their insightful suggestions significantly contributed to improving the quality, clarity, and overall presentation of this manuscript from its initial submission to its final version.

Funding Statement : Not applicable.

Ethics Approval Statement

This article does not involve any studies with human participants or animals performed by any of the authors. Therefore, ethical approval was not required for this study.

Informed Consent Statement

Not applicable, as this study did not involve human participants.

Clinical Trial Registration

Not applicable.

REFERENCES

[1] Y. Caro, A. Hansberg, and M. A. Henning,"Fair domination in graphs," Discrete Mathematics, vol. 312, no. 19, pp. 2905–2914, 2012. https://doi.org/10.1016/j.disc.2012.05.024.

[2] G. Chartrand, L. Lesniak, and P. Zhang, Graphs and Digraphs, 6th ed. Boca Raton, FL, USA: CRC Press, 2015.

[3] T. W. Haynes, S. T. Hedetniemi, and P. J. Slater,Fundamentals of Domination in Graphs. New York, NY, USA: Marcel Dekker, 1998.

[4] T. G. Jayasree and R. Rajamani Iyer,"1-Fair Dominating Sets in Some Classes of Graphs," International Journal of Advanced Trends in Computer Science and Engineering, vol. 8, no. 5, pp. 2438–2441, 2019. https://doi.org/10.30534/ijatcse/2019/85852019

[5] G. Navamani, S. Kalaivani, S. Robinson Chellathurai, and S. K. Venkatachalapathy, "Fair Bondage Number—Analyzing the Changes in Fair Domination," IAENG International Journal of Applied Mathematics, vol. 55, no. 1, pp. 136–142, 2025.

[6] E. Sampathkumar, "The Global Domination Number of a Graph," Journal of Mathematical and Physical Sciences, vol. 23, pp. 377–385, 1989.