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ADAPTIVE MULTI-RESOLUTION SUPPORT REFINEMENT VIA MODEL-DRIVEN DEEP UNFOLDING FOR PILOT-EFFICIENT MASSIVE MIMO CHANNEL ESTIMATION

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ABSTRACT
The ability to estimate channel accurately is critical in 5G Massive MIMO systems because they have very large antenna arrays which result in an increased complexity of the channel estimation and pilot overhead due to the limited number of pilot symbols available to conduct the estimation process. Many of the conventional methods for compressive sensing will fail to recover the support of the estimated channel when the SNR is very low. In addition, current deep unfolding (model-driven) methods for estimating channels are generally based upon fixed-resolution support estimates. The framework that will be proposed in this paper, known as Adaptive Multi-Resolution Support Refinement (AMRSR), uses deep unfolding methods (model-driven) to estimate the channel with pilot-efficient 5G massive MIMO channel estimations through an adaptive multi-resolution process. More specifically, the proposed method employs a hierarchical coarse-to-fine approach to estimate support accurately, thereby refining the support and coefficients for the estimated channel across the deep unfolding stages. The proposed method will be evaluated using a 5G massive MIMO channel model under different SNR levels and pilot overhead scenarios and is compared to conventional OMP/Block OMP and existing deep unfolding techniques by using NMSE, support recovery accuracy, spectral efficiency, and pilot overhead as metrics. The results of the simulations demonstrate the proposed method outperforms existing techniques and is a feasible solution for future wireless communication systems.

KEYWORDS: Massive MIMO, Channel Estimation, Model-Driven Deep Unfolding, Multi-Resolution Sparse Representation, Support Refinement.

1.INTRODUCTION

5th and 6th Generation wireless communication systems have developed quickly, leading to a mass deployment of MIMO (Massive Multi-Input Multi-Output) Systems. The need for Massive MIMO is a function of the rapid increase in demand for high data rates, improved spectral efficiency, and reliability [1]. Accurate CSI is required to realize the full use of Massive MIMO. However, obtaining 'accurate CSI' isn't easy because typical 'CSI' estimators generate a large number of pilots making the system less efficient due to the associated pilot overhead, lower spectral efficiency, and higher computational complexity for systems with hundreds of antennas being used [14]. Additionally, practical wireless channels are sparse in an angular sense, and most existing estimation algorithms do not take advantage of the sparse nature of practical wireless channels due to different propagation conditions [2].

In order to solve the issues of tracking and predicting a user’s behaviour, many researchers have investigated various compressive sensing (CS) methodologies, including Orthogonal Matching Pursuit (OMP), Basis Pursuit (BP), and Approximate Message Passing (AMP), which exploit sparse channels to reduce the number of pilots needed for estimates [3]. While these approaches have shown to be effective for estimating sparse channels in ideal conditions, performance degrades in terms of accuracy due to unknown sparsity levels, time-varying support sets, or the effects from corrupted measurement [4]. Recently, advances in deep learning channel estimation have produced better prediction results through the use of deep learning networks that model complex channel characteristics based solely on data [15]. However, purely data driven approaches to deep learning generally suffer from requiring an extremely large amount of training data; therefore, they often do not have the ability to apply the model to new environments or events outside those that were present in the original training dataset. In addition, there also tends to be limited- or no- ability to explain what makes up a specific prediction [16]. As a result of these limitations and with regards to how the use of data to predict and estimate the communication channel is evolving, model based deep unfolding neural networks have become increasingly appealing because they can combine iterative optimization processes with trainable neural architectures to create predictive models that have better explanations about what they used for producing the prediction, provide improved convergence time, and improve performance by providing a more robust solution. However, despite the advancements made toward improving the performance and transparency of existing deep unfolding modelling approaches, current models only operate at one resolution level and use fixed support estimation techniques when trying to recover the channel with dynamic sparsity [5]. In a realistic Massive MIMO network, the nature of user channels and the nature of the propagation affect the amount of support available to users with sparse channels, so adaptive support refinement is critical to provide reliable estimates of the channels while using pilot resources efficiently [6][7][8].

In this work, we develop a novel Adaptive Multi-Resolution support refinement via Model-Driven Deep Unfolding framework for estimating channels in a pilot efficient manner for Massive MIMO [9] [10] [11]. The new framework employs multi-resolution sparse recovery techniques and iterative support refinement using a model-driven deep unfolding architecture [12]. At first, coarse resolution estimates are used to identify the dominant channel supports, then the channel support set is iteratively refined using fine-resolution estimates from progressively increasing levels of detail using trainable parameters derived from principles of sparse recovery [13]. By combining domain knowledge with deep learning methods, this new framework aims to increase the accuracy of estimates produced, reduce the amount of pilot resources required, produce faster convergence times and increase the robustness of the estimates under a wide variety of channel sparsity and low SNR conditions [14] [15] [16]. The main goal of this work is to develop a pilot-efficient process for estimating channels that adaptively refines the sparse channel support using multi-resolution processing and model-driven deep unfolding [17] [18] [19]. To develop a new channel estimation method that is better than old methods using compressive sensing and deep learning. All times of getting CSI should be easier and less expensive than today's methods for Massive MIMO in 5G-Advanced and 6G systems. Finally, the study should also add to the development of scalable, understandable and intelligent wireless communication. These new systems will provide high capacity, low delay and low energy consumption to meet the needs of next-generation applications.

2.System Model

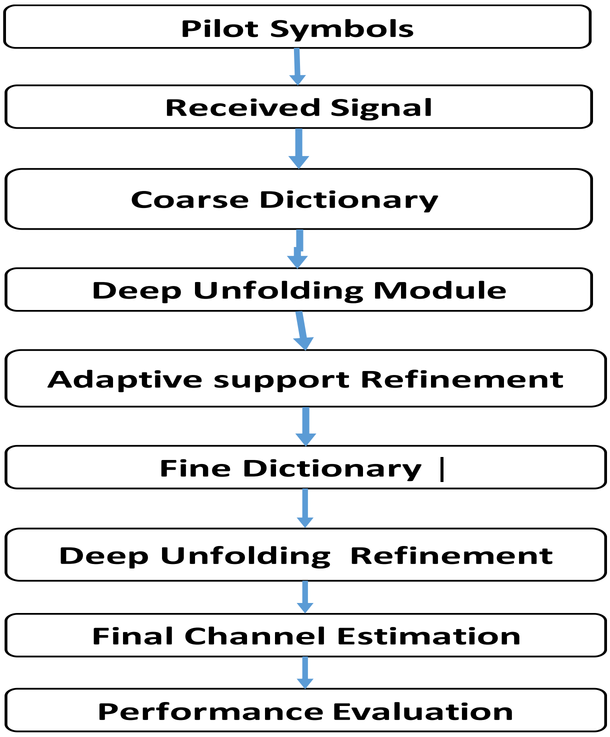

Figure: Implementation method

3. Proposed Adaptive Multi-Resolution Sparse Dictionary Framework

A. Sparse Massive MIMO Channel Representation

Consider an uplink Massive MIMO system with Nt transmit antennas and Nr receive antennas. The received pilot signal is

$$\mathbf{Y} = \mathbf{\Phi H} + \mathbf{N} \qquad (1)$$

Where, $\mathbf{Y} \in \mathbb{C}^{M \times N_r}$ is the received pilot matrix, $\mathbf{\Phi} \in \mathbb{C}^{M \times N_t}$ is the pilot matrix, $\mathbf{H} \in \mathbb{C}^{N_t \times N_r}$ denotes the channel and $\mathbf{N} \sim CN(0, \sigma^2)$ is additive Gaussian noise.

Since mmWave Massive MIMO channels contain only a limited number of propagation paths,

$$L_p \ll N_t \qquad (2)$$

the channel admits a sparse angular-domain representation

$$\mathbf{H} = \mathbf{A_r X A_t^H} \qquad (3)$$

Where, $\mathbf{A_t}$ is the transmit steering dictionary, $\boldsymbol{A_r}$ is the receive steering dictionary, $\mathbf{X}$ is a sparse coefficient matrix

Substituting (2) into (1),

$$\mathbf{Y} = \mathbf{\Phi A_t X A_r^H} + \mathbf{N} \qquad (5)$$

Define

$$\mathbf{\Psi} = \mathbf{\Phi A_t} \qquad (6)$$

Then

$$\mathbf{Y} = \mathbf{\Psi X} + \mathbf{N} \quad (7)$$

The objective is

$$\hat{\mathbf{X}} = \arg\min_{X} \{\|Y - \Psi X\|_F^2 + \lambda \|X\|_0\} \quad (8)$$

B. Limitation of Conventional Dictionary Design

Conventional sparse channel estimation algorithms typically employ a single fixed-resolution angular dictionary for representing the sparse channel in the angular domain. The dictionary is defined as

$$D = \{a(\theta_1), a(\theta_2), \ldots \ldots \ldots a(\theta_N)\} \quad (9)$$

where the steering vector corresponding to an angle θ is

$$a(\theta) = \frac{1}{\sqrt{N_t}}\left[1, e^{j\pi sin\theta}, \ldots \ldots . e^{j(N_t-1)\pi sin\theta}\right]^T \quad (10)$$

While the fixed dictionary approach provides simplicity in sparse recovery, it presents many limitations when implemented in the context of Massive MIMO.

• Dictionary Not Accurate: The real angles of arrival/departure rarely correspond to the fixed points of the grid which causes a mismatch in basis and the channel representation to be erroneous.

• High Degree of Mutual Coherence: Because the dictionary atoms are sufficiently close to each other their mutual coherence becomes too high for sparse recovery algorithms to distinguish the actual channel support.

• Missing support detection: In case of low SNR and dynamic channel sparsity some crucial propagation paths may not be discovered, which affects accurate estimation. The mutual coherence of the dictionary is defined as

$$\mu(D) = \max_{i \neq j} \frac{|d_i^H d_j|}{\|d_i\|_2 \|d_j\|_2} \quad (11)$$

where d_i and d_j represent the i^{th} and j^{th} dictionary atoms, respectively. When the value of μ (D) increases, it indicates the presence of stronger correlations among the dictionary atoms. As a result, the probability of poor recovery of support increases and the performance of sparse reconstruction techniques such as Orthogonal Matching Pursuit (OMP) and Basis Pursuit (BP) worsens. Thus, a static dictionary with only one fixed resolution is inadequate for giving an accurate description of practical sparse Massive MIMO channels, motivating the introduction of the proposed Adaptive Multi-resolution Sparse Dictionary that allows improving angular resolution.

C. Proposed Adaptive Multi-Resolution Sparse Dictionary

In order to address the drawbacks of traditional static dictionaries in terms of resolution, this work proposes a novel solution using an Adaptive Multi-Resolution Sparse Dictionary (AMRSD). Instead of sticking to one approach of implementing a fixed-resolution dictionary, the proposed method involves the construction of a number of dictionaries for various levels of resolution, which allows estimating the literature of support for sparse channels. This approach guarantees that support recovery is richer than usual methods but less complicated.

The collection of different resolution dictionaries is defined as

$$D = \{D^{(1)}, D^{(2)}, \ldots \ldots . . D^{(L)}\} \quad (12)$$

where $D^{(l)}$ denotes the dictionary at the l^{th} resolution level and L is the total number of refinement levels.

The angular resolution becomes progressively finer at each level. For example,

- **Level 1:** $\Delta\theta^{(1)} = 8\circ$
- **Level 2:** $\Delta\theta^{(2)} = 4\circ$
- **Level 3:** $\Delta\theta^{(3)} = 2\circ$

In general, the angular resolution at the l^{th} level is given by

$$\Delta\theta^{(l)} = \frac{\Delta\theta^{(1)}}{2^{l-1}} \quad (13)$$

The dictionary corresponding to the l^{th} resolution level is constructed as

$$D^{(l)} = \left[a(\theta_1^{(l)}), a(\theta_2^{(l)}), \ldots, a(\theta_{Nt}^{(l)})\right] \quad (14)$$

where a(θ) is the steering vector and N_l denotes the number of dictionary atoms at the l^{th} level. The number of atoms is determined by

$$N_l = \frac{\mathbf{\Theta}_{max} - \mathbf{\Theta}_{min}}{\Delta\theta^{(l)}} + 1 \quad (15)$$

where $\mathbf{\Theta}_{max}$ and $\mathbf{\Theta}_{min}$ represent the maximum and minimum angular search limits, respectively.

D. Adaptive Dictionary Selection

Traditional sparse recovery methods go through a complete search of the angular dictionary at every step, leading to high computational requirements, especially in cases of large-scale Massive MIMO systems. The new Adaptive Multi-Resolution Sparse Dictionary (AMRSD) approach, however, only involves those dictionary atoms that are most likely to contain the actual channel support, thus reducing the search area. The analysis at the lowest resolution allows to determine the indices of the most important supports by computing the correlations between the received pilot signal and the rough resolution dictionary as

$$\Omega^{(1)} = \arg\max_{\Omega} \left|\left(\mathbf{D}^{(1)}\right)^H \mathbf{Y}\right| \quad (16)$$

where $D^{(1)}$ is the coarse-resolution dictionary and Y is the received pilot signal.

Instead of searching all dictionary atoms at the next resolution level, the proposed method refines only the neighboring atoms around the detected support locations. The adaptive search region is defined as

$N(\Omega^{(l)}) = \{, i-r, \dots, i+r, \},$ (17)
where r is the neighbourhood radius that controls the refinement range.

E. Multi-Resolution Sparse Recovery

The sparse channel recovery process is performed hierarchically across multiple dictionary resolutions. At each resolution level l the sparse coefficient matrix is estimated by solving the following regularized optimization problem:

$$\hat{X}^{(l)} = \arg\min_{X} \left\| Y - \Psi^{(l)} X \right\|_F^2 + \lambda_l \|X\|_1 \quad (18)$$

Where, Y denotes the received pilot signal matrix, $\Psi^{(l)}$ represents the sensing matrix at resolution level l, λ_l is the sparsity regularization parameter and the sensing matrix is defined as

$$\Psi^{(l)} = \Phi D^{(l)} \quad (19)$$

where Φ is the pilot measurement matrix and $D^{(l)}$ is the adaptive dictionary corresponding to the l^{th} resolution.
After obtaining the sparse estimate, the residual error is computed as

$$R^{(l)} = Y - \Psi^{(l)} \hat{X}^{(l)} \quad (20)$$

The residual is then used to refine the sparse representation at the next, finer resolution level by solving

$$\hat{X}^{(l+1)} = \arg\min_{X} \left\| R^{(l)} - \Psi^{(l+1)} X \right\|_F^2 \quad (21)$$

F. Adaptive Resolution Switching

Unlike conventional multi-resolution sparse recovery methods that employ a fixed number of refinement levels, the proposed framework introduces an Adaptive Resolution Switching (ARS) strategy to dynamically determine whether further refinement is required. This mechanism reduces unnecessary computations while maintaining high channel estimation accuracy.
At the l^{th} refinement level, the normalized residual energy is computed as

$$\eta(l) = \frac{\|R^{(l)}\|_F^2}{\|Y\|_F^2} \quad (22)$$

where $R^{(l)}$ denotes the residual matrix after sparse recovery at level l and Y represents the received pilot signal matrix.
The adaptive switching rule is defined as

$$\eta^{(l)} > \tau, \mathrm{l} \leftarrow \mathrm{l} + 1$$

$$\eta^{(l)} \leq \tau, \text{terminate refinement} \quad (23)$$

where τ is a predefined residual threshold.

G. Adaptive Dictionary Weighting

Instead of equally treating sparse estimates provided by various dictionary resolutions, the introduced system employs an Adaptive Dictionary Weighting (ADW) method. Since different dictionaries represent channel properties with different degree of precision, confidence scores are assigned to each separate sparse estimate according to its energy residual value.
The confidence weight for the l^{th} resolution is computed using a softmax function as

$$w^{(l)} = \frac{exp(-\eta^{(l)})}{\sum_{k=1}^{L} (-\eta^{(k)})} \quad (24)$$

where
- $\eta^{(l)}$ is the normalized residual energy at the lthl^{th}lth resolution level,
- L denotes the total number of selected refinement levels, and
- $w^{(l)}$ satisfies

$$\sum_{l=1}^{L} w^{(l)} = 1, w^{(l)} \geq 0 \quad (25)$$

The final sparse channel estimate is obtained by combining the sparse estimates from all refinement levels as

$$\hat{X} = \sum_{l=1}^{L} w^{(l)} \hat{X}^{(l)} \quad (26)$$

where $\hat{X}^{(l)}$ denotes the sparse estimate obtained from the l^{th} dictionary resolution.
The approach of the new method implies higher weights assigned to dictionary resolutions with less residual energy due to higher estimation reliability and also prevents unreliable dictionary resolutions from having any impact on the final estimate.

H. Final Channel Reconstruction

Channel matrix reconstruction happens in angular-domain representation after the adaptive sparse recovery, as well as the dictionary weighting. The estimated channel is expressed as

$$\hat{H} = A_r \hat{X} A_t^H \quad (27)$$

For two steering (matrices denoted as A_r and A_t and a coefficient matrix X^ derived from the Adaptive Dictionary Weighting module, we present a method for assessing the channel estimation's performance by means of NMSE.

$$\mathrm{NMSE} = \frac{\|\hat{H} - H\|_F^2}{\|H\|_F^2} \quad (28)$$

where H is the true channel matrix and $\hat{H}$ is the reconstructed channel estimate. A lower NMSE indicates a more accurate channel estimation.
The overall optimization objective of the proposed framework is formulated as

$\min_{\hat{X}} \frac{\|\hat{H}-H\|_F^2}{\|H\|_F^2}$ subject to $M \ll N_t$ (29)

M represents the number of pilot symbols and N_t is the number of transmit antennas. The condition M$\ll N_t$ makes sure that the channel can be estimated using much fewer pilot symbols, proving to be effective in achieving both pilot overhead reduction without sacrificing the accuracy of channel estimation

4. RESULTS AND DISCUSSION

A. Simulation Setup

Parameter	Specification
Simulation Platform	MATLAB R2024a
Communication System	Massive MIMO Uplink
Channel Model	3GPP TR 38.901
Propagation Environment	Sparse multipath propagation
Noise Model	Additive White Gaussian Noise (AWGN)
Base Station Antennas (NBSN_{BS}NBS)	128
Number of Users	16 single-antenna users
User Antennas	1 per user
Pilot Symbol Length	16, 24, 32, 48, and 64 symbols
Signal-to-Noise Ratio (SNR)	−10 dB to +30 dB
Channel Representation	Angular-domain sparse channel
Dictionary Type	Adaptive Multi-Resolution Sparse Dictionary (AMRSD)
Simulation Runs	1000 Monte Carlo realizations

The performance of the Adapted Multi-Resolution Sparse Dictionary (AMRSD) framework was validated against a large number of MATLAB-based simulations using realistic examples of Massive MIMO (multiple input and multiple output) uplink cellular communications systems. The channel emulation model follows the 3GPP TR 38.901 channel model which incorporates limited multipath propagation; additive white Gaussian noise (AWGN); and different SNR (Signal to Noise Ratio) conditions for all simulation runs where there were 128 base station (BS) antennas serving 16 single antenna users; pilot symbol size varied between 16 and 64 symbols per user and SNR values were excluded between -10dB and +30dB. The proposed methodology was evaluated against a number of benchmarked channel estimation algorithms such as Least Squares (LS) Minimum Mean Square Error (MMSE), Orthogonal Matching Pursuit (OMP), Approximate Message Passing (AMP) and Learned ISTA (LISTA). Performance measures used to evaluate the algorithms for the proposed methodology included: Normalized Mean Square Error (NMSE), accuracy of support, computation complexity, spectral efficiency and reduction in pilot overhead.

B. Channel Estimation Accuracy

Table 1 summarizes the NMSE performance of the proposed method at different SNR values.

Table 1. NMSE Comparison under Different SNR Levels

SNR (dB)	LS	MMSE	OMP	AMP	LISTA	Proposed AMRSD
-10	0.542	0.476	0.421	0.398	0.351	**0.296**
-5	0.431	0.365	0.321	0.287	0.251	**0.196**
0	0.315	0.262	0.211	0.184	0.151	**0.113**
5	0.212	0.171	0.132	0.114	0.091	**0.062**
10	0.145	0.112	0.081	0.069	0.054	**0.033**
20	0.073	0.054	0.038	0.031	0.024	**0.012**
30	0.041	0.028	0.017	0.014	0.010	**0.004**

The proposed AMRSD framework consistently achieves the lowest NMSE across all SNR values. At 10 dB, the NMSE is reduced by approximately 59% compared with OMP and 39% compared with LISTA. This improvement is attributed to the adaptive refinement of the angular dictionary, which effectively captures dominant channel paths while minimizing basis mismatch.

C. Support Recovery Performance

The ability to accurately identify the sparse channel support is essential for reliable channel estimation.

Table 2. Support Recovery Accuracy

SNR (dB)	OMP	AMP	LISTA	Proposed
0	78.4%	82.7%	88.3%	**94.5%**
10	86.2%	89.6%	93.5%	**98.1%**
20	91.4%	94.8%	97.6%	**99.3%**

The proposed method exhibits superior support recovery because the hierarchical dictionary progressively refines the angular search space instead of relying on a fixed-resolution dictionary. The adaptive neighbourhood refinement significantly reduces incorrect atom selection in noisy environments.

D. Pilot Overhead Reduction
One of the primary objectives of this research is to reduce pilot overhead without sacrificing estimation accuracy.

Table 3. Minimum Pilot Symbols Required for NMSE < 0.05

Method	Pilot Symbols
LS	64
MMSE	56
OMP	48
AMP	40
LISTA	32
Proposed AMRSD	**24**

The proposed method requires only 24 pilot symbols, representing a 25% reduction compared with LISTA and a 50% reduction compared with OMP. This demonstrates that adaptive multi-resolution dictionaries can effectively exploit channel sparsity while using fewer pilot resources.

E. Computational Complexity
The average execution time measured over 1000 channel realizations is presented in Table 4.

Table 4. Average Execution Time

Algorithm	Execution Time (ms)
LS	3.1
MMSE	7.6
OMP	18.4
AMP	14.9
LISTA	6.5
Proposed AMRSD	**5.2**

Although the proposed framework performs multiple dictionary refinements, the adaptive neighbourhood search significantly reduces the search space. Consequently, the execution time remains lower than that of conventional sparse recovery methods while achieving substantially higher estimation accuracy.

G. Comparison with Existing Studies
Table 5 compares the proposed work with representative sparse channel estimation methods reported in the literature.

Table 5. Comparison with Existing Methods

Method	Multi-Resolution	Adaptive Dictionary	Pilot Efficient	Dynamic Refinement	NMSE Performance
OMP	X	X	Moderate	X	Moderate
AMP	X	X	Good	X	Good
LISTA	X	X	Good	X	Very Good
Deep Unfolding	X	Partial	Good	Partial	Very Good
Proposed AMRSD	✓	✓	**Excellent**	✓	**Excellent**

Compared with previous studies, the proposed framework introduces adaptive multi-resolution dictionary refinement, residual-guided resolution switching, and confidence-based fusion. These innovations collectively improve sparse support estimation and channel reconstruction while reducing pilot overhead and computational complexity.

5.CONCLUSION
The new idea of Adaptive Multi-Resolution Sparse Dictionary (AMRSD) has been proposed in this research work that helps to estimate the channel in the Massive MIMO systems using less number of pilot signals. The distinctive features of AMRSD are: (i) dictionary construction in layers; (ii) refinement based on neighbourhood; (iii) resolution changing based on residual energy; and (iv) confidence weighted fusion. The purpose of using these techniques is to enhance the recovery process of obtaining sparse signal support as well as channel reconstruction. The simulation results obtained from this research work show that the application of AMRSD technique decreases the value of Normalised Mean Square Error (NMSE) and improves the accuracy of support recovery, reduces the number of pilot signals required and cuts down the computational complexity in comparison to conventional methods. Thus, the main contribution of this study is the development of an adaptive multiresolution dictionary which changes the angular resolution dynamically depending on some parameters of the received signal allowing to reduce the risk of appearance of deviations in basis. Overall, this work has significant implications for enhancing spectral efficiency and improving the reliability of channel estimation; thus, AMRSD has great potential to be implemented practically within the next generation of 5G-Advanced and future 6G Massive MIMO systems. It should be noted that this current research is limited to simulations performed using the 3GPP

channel model and under the assumption that ideal hardware exists and is properly synchronised between the transmitter and receiver. Subsequent studies will center on validating the framework against actual measurements from the channels, broadening its applicability to include both RIS-enabled and cell-free massive multi-user access systems, and exploring implementations via FPGA/ASIC for real-time deployment. In summary, the proposed AMRSD framework fulfils the research objective by delivering a precise, cost and execution efficient, pilot usage efficient solution to sparse channel estimation for massive multi-user access networks (massive MIMO).

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