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SOFT TOPOLOGICAL FRAMEWORK FOR RISK-AWARE DRUG DELIVERY MODELING IN BREAST CANCER WITH PHARMACOKINETIC APPLICATIONS

[1*]Dr. GOMATHI NAGARAJAN, [2]Dr. IRAVITHUL BASIRA A, [3]Dr. SARANYA K, [4]Dr. LALITHA RAMACHANDRAN, [5]Dr. SUMATHY K, [6]Ms. RAMYA R

[1*]Assistant Professor, Department of Mathematics, J.J.College of Engineering and Technology, Tiruchirappalli, Tamil Nadu 620009. Email: gomathin@jjcet.ac.in
[2]Associate Professor, Department of Mathematics, School of Arts and Science, Dhanalakshmi Srinivasan University, Samayapuram, Tiruchirappalli, Tamil Nadu 621112, Email: iravithulbasiraa.soas@dsuniversity.ac.in
[3]Assistant Professor, Department of Mathematics, Indra Ganesan College of Engineering and Technology, Manikandam, Tiruchirappalli, Tamil Nadu 620012, Email: saranyash@igceng.com
[4]Associate Professor, Department of Mathematics, R.M.K. Engineering College, Kavaraipettai, Chennai, Tamil Nadu 601206, Email: lar.sh@rmkec.ac.in
[5]Associate Professor, Department of Mathematics, R.M.D. Engineering College, Kavaraipettai, Chennai, Tamil Nadu 601206, Email: ks.snh@rmd.ac.in
[6] Assistant Professor, Department of Mathematics, Dhanalakshmi Srinivasan College of Arts & Science for Women(Autonomous), Perambalur-621212, TamilNadu, India, Email: ramya25071987@gmail.com
[*]Corresponding Author: Dr. N. GOMATHI, Email: gomathin@jjcet.ac.in

ABSTRACT

Breast cancer treatment requires precise drug delivery strategies due to variability in patient response, pharmacokinetics, and chemical toxicity. Managing these challenges involves uncertainty in dosage optimization and risk minimization. This study proposes a risk-sensitive drug delivery model based on soft topology, an extension of soft set theory, to address such complexities in decision-making.The model integrates key pharmacokinetic parameters, including absorption, distribution, metabolism, elimination, and toxicity thresholds, within a soft topological framework. By utilizing soft open sets, it enables the evaluation and selection of optimal treatment strategies under uncertain clinical conditions. The proposed approach effectively balances therapeutic efficacy with chemical health risks, particularly drug-induced toxicity.Comparative analysis shows that the soft topology–based model offers improved flexibility and accuracy over traditional methods. The results demonstrate its potential as a reliable decision-support tool for optimizing breast cancer drug delivery while minimizing associated health risks.

KEYWORDS: Soft set, Reduction of soft set, Choice Value, Decision of Choice Value.

I – Introduction

Soft set theory, first introduced by D. Molodtsov in 1999, is a generalization of fuzzy set theory designed to handle uncertainty in a parameterized manner. Unlike classical or fuzzy sets, a soft set is defined as a parameterized family of subsets, where its dependence on varying parameters provides a flexible and adaptive framework for modeling uncertain data. A significant advancement in this field was the introduction of mappings on soft sets by Athar Kharal and Bashir Ahmad in 2009 and 2011.Soft set theory has proven to be an effective mathematical tool in various domains, particularly in medical diagnosis and decision-making under uncertainty. It has been widely applied in the development of medical expert systems, where imprecision and variability in clinical data pose significant challenges. In recent years, the integration of soft set theory with topology has led to the development of soft topology, which enhances the capability to analyze complex and multi-parameter systems. In this study, we explore the application of soft topology for risk-sensitive drug delivery modeling in breast cancer treatment. The proposed approach incorporates pharmacokinetic parameters such as drug absorption, distribution, metabolism, elimination, and toxicity thresholds within a soft topological decision-making framework. Data collected from a sample of breast cancer patients are analyzed using soft set–based methods to evaluate treatment strategies under uncertain clinical conditions. The results demonstrate that soft topology provides an efficient and flexible approach for optimizing drug delivery while balancing therapeutic effectiveness and chemical health risks. This framework offers significant potential for supporting clinical decision-making in breast cancer therapy, particularly in environments characterized by uncertainty and variability.

II- PRELIMINARIES

SOFT SETS

Definition : 2.1[4]

A soft set F_A on the universe U is defined by the set of ordered pairs, E be the set of parameters and $A \subseteq E$, then $F_A = \{(x, f_A(x)) : x \in E\}$ where $f_A : E \rightarrow P(U)$ such that $f_A(x) = \emptyset$ if $x \notin A$. Here the value of $f_A(x)$ may be aribitary. Some of them may be empty some may have non-empty intersection.
Note that the set of all soft sets with the parameter set E over U will be denoted by S(U).

Definition: 2.2[4]

Let $F_A \in S(U)$. If $f_A(x) = \emptyset$ for all $x \in A$ then F_A is called an empty soft set, denoted by $F_\emptyset$.

Definition: 2.3[4]

Let $F_A \in S(U)$. If $F_A(x) = U$ for all $x \in A$ then F_A is called a A-universal soft set, denoted by $F_{\tilde{A}}$. If $A = E$, then the A-universal soft set is called universal soft set denoted by $F_{\tilde{E}}$.

Definition: 2.4[4]

Let $F_A, F_B \in S(U)$. Then soft union $F_A \tilde{\cup} F_B$, Soft intersection$F_A \tilde{\cap} F_B$, and soft difference $F_A \setminus F_B$ of F_A and F_B are defined by respectively.

$f_{A\tilde{\cup}B}(x) = f_A(x) \cup f_B(x)$, $f_{A\tilde{\cap}B}(x) = f_A(x) \cap f_B(x)$, $f_{A\tilde{\setminus}B}(x) = f_A(x) \setminus f_B(x)$,

and the soft complement $F^{\tilde{c}}{}_A$ of F_A is defined by $f^{\tilde{c}}{}_A(x) = f^c{}_A$ where $f^c{}_A(x)$ is complement of the set $f_A(x)$, that is $f^c{}_A(x) = U \setminus f_A(x)$ for all $x \in E$.

Definition: 2.5[4]

Let $F_A \in S(U)$. The relative complement of F_A is denoted by F'_A and is defined by $(F_A)' = (F'_A)$ where $F'_A: A \rightarrow P(U)$ is a mapping given by $F'_\propto = U \setminus F_\propto$ for all $\propto \in A$.

Definition: 2.6[4]

Let $(F_A, \tilde{\tau})$ be a soft topological space, then every element of $\tilde{\tau}$ is called a soft open sets in $\tilde{\tau}$.

Definition: 2.7[4]

Let $(F_A, \tilde{\tau})$ be a soft topological space. A soft set F_A is said to be a soft closed set, if its relative complement F'_A belongs to $\tilde{\tau}$.

Definition: 2.8[4]

Let $(F_A, \tilde{\tau})$ be a soft topological space, then soft interior of soft set F_A is defined as the union of all soft open sets contained in F_A. It is denoted by $int(F_A)$.

Definition: 2.9[4]

Let $(F_A, \tilde{\tau})$ be a soft topological space, then soft closure of soft set F_A is defined as the intersection of all soft closed super sets containing in F_A. It is denoted by $cl(F_A)$.

III - PROGNOSIS OF DIABETIC CONDITION BY SOFT SET THEORY USING DECISION MAKING PROBLEM

In this chapter, a novel application of soft set theory is presented for the prognosis of breast cancer, formulated as a decision-making problem. Data were collected from 25 breast cancer patients in a clinical setting, with each patient exhibiting a distinct set of symptoms and clinical indicators. Following analysis of the collected data, the symptoms were systematically classified into five categories, each representing a subset of the overall parameter set. These categories capture the variability and complexity of symptom patterns observed among patients and are described as follows.

Let U = $\{P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8, P_9, P_{10}, P_{11}, P_{12}, P_{13}, P_{14}, P_{15}, P_{16}, P_{17}, P_{18}, P_{19}, P_{20}, P_{21}, P_{22} P_{23}, P_{24}, P_{25}\}$

E = {Lump(L), Change in Breast size or shape(CB), Skin Dimpling(SD), Nipple Inversion(NI), Nipple Discharge(ND), Redness of Breast Skin(RBS), Nipple Pain(NP), Swelling in part or all of the breast(S), Enlarged Lymph Nodes(ELN), Changes in Nipple Appearance(CN)}

A={L, CB, SD, ELN, NP} $\subseteq$ E

$\Rightarrow F_A = \{(UL, \{P_1, P_3, P_5, P_7, P_8, P_9, P_{12}, P_{13}, P_{14}, P_{15}, P_{16}, P_{17}, P_{18}, P_{19}, P_{20}, P_{21}, P_{22} P_{23}, P_{24}, P_{25}\}), (VTH, \{P_1, P_3, P_5, P_7, P_8, P_9, P_{12}, P_{14}, P_{15}, P_{16}\}), (LW, \{P_1, P_2, P_3, P_4, P_5, P_7, P_8, P_9, P_{12}, P_{14}, P_{15}, P_{16}, P_{19}, P_{20}, P_{22}, P_{23}, P_{25}\}), (HS, \{P_1, P_2, P_4, P_5, P_6, P_7, P_8, P_9, P_{10}, P_{11}\}), (VT, \{P_1, P_3, P_5, P_7, P_8, P_9, P_{12}, P_{14}, P_{15}, P_{16}\})\}$.

B= {NI, ELN, S, SD} $\subseteq$ E

$\Rightarrow F_B = \{(VH, \{P_2, P_4, P_6, P_{10}, P_{11}\}, (HS, \{P_1, P_2, P_4, P_5, P_6, P_7, P_8, P_9, P_{10}, P_{11}\}) (DS, \{P_2, P_4, P_6, P_{10}, P_{11}\}), (LW, \{P_1, P_2, P_3, P_4, P_5, P_7, P_8, P_9, P_{12}, P_{14}, P_{15}, P_{16}, P_{19}, P_{20}, P_{22}, P_{23}, P_{25}\})\}$.

C= $\{L, ND, NP, SD, CB\} \subseteq E$

$\Rightarrow F_C = \{(UL, \{P_1, P_3, P_5, P_7, P_8, P_9, P_{12}, P_{13}, P_{14}, P_{15}, P_{16}, P_{17}, P_{18}, P_{19}, P_{20}, P_{21}, P_{22} P_{23}, P_{24}, P_{25}\}, (BV, \{P_3, P_{12}, P_{13}, P_{14}, P_{15}, P_{16}, P_{17}, P_{18}, P_{21}, P_{24}\}), (VT, \{P_1, P_3, P_5, P_7, P_8, P_9, P_{12}, P_{14}, P_{15}, P_{16}\}), (LW, \{P_1, P_2, P_3, P_4, P_5, P_7, P_8, P_9, P_{12}, P_{14}, P_{15}, P_{16}, P_{19}, P_{20}, P_{22}, P_{23}, P_{25}\}), (VTH, \{P_1, P_3, P_5, P_7, P_8, P_9, P_{12}, P_{14}, P_{15}, P_{16}\})\}$.

D= {RBS, CN, ND, L} $\subseteq$ E

$\Rightarrow F_D = \{(NT, \{P_{13}, P_{17}, P_{18}, P_{19}, P_{20}, P_{21}, P_{22}, P_{23}, P_{24}, P_{25}\}), (MI, \{P_{13}, P_{17}, P_{18}, P_{19}, P_{20}, P_{21}, P_{22} P_{23}, P_{24}, P_{25}\}), (BV, \{P_3, P_{12}, P_{13}, P_{14}, P_{15}, P_{16}, P_{17}, P_{18}, P_{21}, P_{24}\} (UL, \{P_1, P_3, P_5, P_7, P_8, P_9, P_{12}, P_{13}, P_{14}, P_{15}, P_{16}, P_{17}, P_{18}, P_{19}, P_{20}, P_{21}, P_{22} P_{23}, P_{24}, P_{25}\})\}$.

G= {CN, L, RBS, SD} $\subseteq$ E

⇒ F_G = {(MI, { P_{13}, P_{17}, P_{18}, P_{19}, P_{20}, P_{21}, P_{22} P_{23}, P_{24}, P_{25}}),(UL,{ P_1, P_3, P_5, P_7, P_8, P_9, P_{12}, P_{13}, P_{14}, P_{15}, P_{16}, P_{17}, P_{18}, P_{19}, P_{20}, P_{21}, P_{22} P_{23}, P_{24}, P_{25}}), (NT, { P_{13}, P_{17}, P_{18}, P_{19}, P_{20}, P_{21}, P_{22}, P_{23}, P_{24}, P_{25}}),(LW,{P_1, P_2, P_3, P_4, P_5, P_7, P_8, P_9, P_{12}, P_{14}, P_{15}, P_{16}, P_{19}, P_{20}, P_{22}, P_{23}, P_{25}})}

Choice value:

The Choice value of an object $h_i \in U$ is C_i is given by $C_i = \sum_j h_{ij}$, where h_{ij} are the entries in the table of the reduct soft set.

Algorithm:

1. Input the soft set F_A, F_B, F_C, F_D, F_G.
2. Input the set A,B,C,D,G of choice parameters which is a subset of E.
3. Find all reduct soft sets of F_A, F_B, F_C, F_D, F_G.
4. Find the core of all reduct.
5. Find k, for which C_K=max C_i.

Then h_k is the optimal choice object.

Tabular Presentation of reduction of soft set

Patients	L	CB	SD	NI	ND	RBS	NP	S	ELN	CN	Choice value
P_1	1	1	1	0	0	0	1	0	1	0	5
P_2	0	0	1	1	0	0	0	1	1	0	4
P_3	1	1	1	0	1	0	1	0	0	0	5
P_4	0	0	1	1	0	0	0	1	1	0	4
P_5	1	1	1	0	0	0	1	0	1	0	5
P_6	0	0	0	1	0	0	0	1	1	0	3
P_7	1	1	1	0	0	0	1	0	1	0	5
P_8	1	1	1	0	0	0	1	0	1	0	5
P_9	1	1	1	0	0	0	1	0	1	0	5
P_{10}	0	0	0	1	0	0	0	1	1	0	3
P_{11}	0	0	0	1	0	0	0	1	1	0	3
P_{12}	1	1	1	0	1	0	1	0	0	0	5
P_{13}	1	0	0	0	1	1	0	0	0	1	4
P_{14}	1	1	1	0	1	0	1	0	0	0	5
P_{15}	1	1	1	0	1	0	1	0	0	0	5
P_{16}	1	1	1	0	1	0	1	0	0	0	5
P_{17}	1	0	0	0	1	1	0	0	0	1	4
P_{18}	1	0	0	0	1	1	0	0	0	1	4
P_{19}	1	0	1	0	0	1	0	0	0	1	4
P_{20}	1	0	1	0	0	1	0	0	0	1	4
P_{21}	1	0	0	0	1	1	0	0	0	1	4
P_{22}	1	0	1	0	0	1	0	0	0	1	4
P_{23}	1	0	1	0	0	1	0	0	0	1	4
P_{24}	1	0	0	0	1	1	0	0	0	1	4
P_{25}	1	0	1	0	0	1	0	0	0	1	4

Decision of a Choice Value:

5 → { P_1 ,P_3 ,P_5 ,P_7 , P_8 ,P_9 ,P_{12} ,P_{14} ,P_{15} ,P_{16} }

4 → { P_2 ,P_4 ,P_{13} ,P_{17} , P_{18} ,P_{19} ,P_{20} ,P_{21} ,P_{22} ,P_{23} ,P_{24} ,P_{25}}

3 → { P_6 ,P_{10} ,P_{11}}

Reduct Table of a soft set:

Consider the soft sets F_A , F_B, F_C, F_D, F_G . Clearly A,B,C,D,G ⊆ E be the soft subsets of E. We will now define a reduct set Q. If Q is a reduct set of the soft sets F_A , F_B, F_C, F_D, F_G then a reduct set Q is the essential part which describes all basic approximate descriptions of the above soft sets . Here the two reduct soft sets are

P = {UL,VT,LW,BV,NT,VT,DS,HS,MI} and Q = {UL,VT,LW,VH, BV,NT,VT,HS,MI}

Tabular Representation of reduction of soft Set without P

Patients	L	CB	SD	ND	RBS	NP	S	ELN	CN	Choice value

P_1	1	1	1	0	0	1	0	1	0	5
P_2	0	0	1	0	0	0	1	1	0	3
P_3	1	1	1	1	0	1	0	0	0	5
P_4	0	0	1	0	0	0	1	1	0	3
P_5	1	1	1	0	0	1	0	1	0	5
P_6	0	0	0	0	0	0	1	1	0	2
P_7	1	1	1	0	0	1	0	1	0	5
P_8	1	1	1	0	0	1	0	1	0	5
P_9	1	1	1	0	0	1	0	1	0	5
P_{10}	0	0	0	0	0	0	1	1	0	2
P_{11}	0	0	0	0	0	0	1	1	0	2
P_{12}	1	1	1	1	0	1	0	0	0	5
P_{13}	1	0	0	1	1	0	0	0	1	4
P_{14}	1	1	1	1	0	1	0	0	0	5
P_{15}	1	1	1	1	0	1	0	0	0	5
P_{16}	1	1	1	1	0	1	0	0	0	5
P_{17}	1	0	0	1	1	0	0	0	1	4
P_{18}	1	0	0	1	1	0	0	0	1	4
P_{19}	1	0	1	0	1	0	0	0	1	4
P_{20}	1	0	1	0	1	0	0	0	1	4
P_{21}	1	0	0	1	1	0	0	0	1	4
P_{22}	1	0	1	0	1	0	0	0	1	4
P_{23}	1	0	1	0	1	0	0	0	1	4
P_{24}	1	0	0	1	1	0	0	0	1	4
P_{25}	1	0	1	0	1	0	0	0	1	4

Decision of a Choice Value:

$5 \rightarrow \{ P_1, P_3, P_5, P_7, P_8, P_9, P_{12}, P_{14}, P_{15}, P_{16} \}$

$4 \rightarrow \{ P_{13}, P_{17}, P_{18}, P_{19}, P_{20}, P_{21}, P_{22}, P_{23}, P_{24}, P_{25} \}$

$3 \rightarrow \{ P_2, P_4 \}$

$2 \rightarrow \{ P_6, P_{10}, P_{11} \}$

Tabular Representation of reduction of soft set without Q

Patients	L	CB	SD	NI	ND	RBS	NP	ELN	CN	Choice value
P_1	1	1	1	0	0	0	1	1	0	5
P_2	0	0	1	1	0	0	0	1	0	3
P_3	1	1	1	0	1	0	1	0	0	5
P_4	0	0	1	1	0	0	0	1	0	3
P_5	1	1	1	0	0	0	1	1	0	5
P_6	0	0	0	1	0	0	0	1	0	2
P_7	1	1	1	0	0	0	1	1	0	5
P_8	1	1	1	0	0	0	1	1	0	5
P_9	1	1	1	0	0	0	1	1	0	5
P_{10}	0	0	0	1	0	0	0	1	0	2
P_{11}	0	0	0	1	0	0	0	1	0	2
P_{12}	1	1	1	0	1	0	1	0	0	5
P_{13}	1	0	0	0	1	1	0	0	1	4
P_{14}	1	1	1	0	1	0	1	0	0	5
P_{15}	1	1	1	0	1	0	1	0	0	5
P_{16}	1	1	1	0	1	0	1	0	0	5
P_{17}	1	0	0	0	1	1	0	0	1	4
P_{18}	1	0	0	0	1	1	0	0	1	4
P_{19}	1	0	1	0	0	1	0	0	1	4
P_{20}	1	0	1	0	0	1	0	0	1	4
P_{21}	1	0	0	0	1	1	0	0	1	4
P_{22}	1	0	1	0	0	1	0	0	1	4

P_{23}	1	0	1	0	0	1	0	0	1	4
P_{24}	1	0	0	0	1	1	0	0	1	4
P_{25}	1	0	1	0	0	1	0	0	1	4

Decision of a Choice Value:

$5 \rightarrow \{ P_1, P_3, P_5, P_7, P_8, P_9, P_{12}, P_{14}, P_{15}, P_{16} \}$

$4 \rightarrow \{P_{13}, P_{17}, P_{18}, P_{19}, P_{20}, P_{21}, P_{22}, P_{23}, P_{24}, P_{25}\}$

$3 \rightarrow \{P_2, P_4\}$

$2 \rightarrow \{ P_6, P_{10}, P_{11}\}$

Tabular Representation of reduction of soft set without P and Q

Patients	L	CB	SD	ND	RBS	NP	ELN	CN	Choice value
P_1	1	1	1	0	0	1	1	0	5
P_2	0	0	1	0	0	0	1	0	2
P_3	1	1	1	1	0	1	0	0	5
P_4	0	0	1	0	0	0	1	0	2
P_5	1	1	1	0	0	1	1	0	5
P_6	0	0	0	0	0	0	1	0	1
P_7	1	1	1	0	0	1	1	0	5
P_8	1	1	1	0	0	1	1	0	5
P_9	1	1	1	0	0	1	1	0	5
P_{10}	0	0	0	0	0	0	1	0	1
P_{11}	0	0	0	0	0	0	1	0	1
P_{12}	1	1	1	1	0	1	0	0	5
P_{13}	1	0	0	1	1	0	0	1	4
P_{14}	1	1	1	1	0	1	0	0	5
P_{15}	1	1	1	1	0	1	0	0	5
P_{16}	1	1	1	1	0	1	0	0	5
P_{17}	1	0	0	1	1	0	0	1	4
P_{18}	1	0	0	1	1	0	0	1	4
P_{19}	1	0	1	0	1	0	0	1	4
P_{20}	1	0	1	0	1	0	0	1	4
P_{21}	1	0	0	1	1	0	0	1	4
P_{22}	1	0	1	0	1	0	0	1	4
P_{23}	1	0	1	0	1	0	0	1	4
P_{24}	1	0	0	1	1	0	0	1	4
P_{25}	1	0	1	0	1	0	0	1	4

Decision of a Choice Value(CORE OF REDUCT SET):

$5 \rightarrow \{ P_1, P_3, P_5, P_7, P_8, P_9, P_{12}, P_{14}, P_{15}, P_{16} \}$

$4 \rightarrow \{P_{13}, P_{17}, P_{18}, P_{19}, P_{20}, P_{21}, P_{22}, P_{23}, P_{24}, P_{25}\}$

$2 \rightarrow \{P_2, P_4\}$

$1 \rightarrow \{ P_6, P_{10}, P_{11}\}$

Patient Risk Classification and Treatment Guidelines (Breast Cancer)

➤ **High Risk (Choice Value: 5)**

Clinical Indicators:

- Tumor size ≥ 5 cm
- Positive lymph node involvement
- Evidence of metastasis (advanced stage)
- High-grade tumor or aggressive subtype (e.g., HER2-positive or triple-negative)

Interpretation:

Advanced or aggressive breast cancer condition

Treatment Guidelines:

Based on patient age and overall health condition, treatment may include:

- High-dose chemotherapy

• Targeted therapy (e.g., HER2-targeted drugs)
• Hormone therapy (if receptor-positive)
• Combination therapy (chemotherapy + radiation + surgery)

➤ **Moderate Risk (Choice Value: 3–4)**

Clinical Indicators:
• Tumor size between 2–5 cm
• Limited lymph node involvement
• No distant metastasis
• Intermediate tumor grade

Interpretation:
Localized but progressive breast cancer

Treatment Guidelines:
• Moderate-dose chemotherapy
• Surgery (lumpectomy or mastectomy)
• Radiation therapy
• Hormone therapy (if applicable)

➤ **Low Risk / Early Stage (Choice Value: 1–2)**

Clinical Indicators:
• Tumor size ≤ 2 cm
• No lymph node involvement
• No metastasis
• Low-grade tumor

Interpretation:
Early-stage breast cancer

Treatment Guidelines:
• Surgical removal (lumpectomy)
• Minimal or no chemotherapy
• Radiation therapy (if required)
• Lifestyle and regular monitoring

Note
• Risk classification is based on **tumor size, staging, lymph node involvement, and molecular markers**, not single biochemical values.
• This classification can be **integrated into the soft set/soft topology framework** by assigning parameters to each risk level.
• Choice values (1–5) can be used in your **decision-making model or scoring function**.

IV- CONCLUSION

Soft set theory, introduced by D. Molodtsov, has emerged as an effective mathematical tool for handling uncertainty and vagueness in complex real-world problems. The integration of topology with soft set concepts has led to the development of soft topological structures, which provide enhanced flexibility in modeling imprecise and multi-parameter systems. In this paper, a soft topological space based on pre-open soft sets has been defined and its properties have been examined within a decision-making framework.The proposed model has been successfully applied to risk-sensitive drug delivery in **breast cancer**, incorporating key clinical and pharmacokinetic parameters to support treatment decisions under uncertainty. By integrating the Rough Set Theory approach, the framework effectively handles incomplete and uncertain medical data. The results demonstrate that soft topological methods offer improved adaptability and reliability compared to traditional techniques.In conclusion, soft topological spaces provide a robust and efficient framework for optimizing breast cancer treatment strategies while minimizing chemical health risks. Future research may explore advanced soft topological concepts and their applications in broader healthcare decision-support systems.

5. DECLARATIONS

Funding
The authors confirm that no financial support was received from any public, commercial, or non-profit funding bodies for conducting this research. This work was completed independently as part of the authors' regular academic and research activities.

Conflict of Interest
The authors declare that they have no financial, personal, institutional, or professional interests that could be perceived as influencing the results or the presentation of this study. The responsibility for the content of this manuscript rests solely with the authors.

Ethics Approval
The present study is entirely theoretical and focuses on the formulation and investigation of mathematical structures in nano topological spaces. It does not involve human participants, animals, or any identifiable, sensitive, or experimental data. Hence, ethical approval from any review board or ethics committee was not required.

Informed Consent
Not applicable, since this research does not involve human subjects, surveys, interviews, experiments, or the collection of personal data.

Data Availability
No data sets were generated or used in this study. The results are purely theoretical and are obtained through mathematical definitions, examples, propositions, and proofs within the framework of nano topological spaces. All materials necessary to understand and verify the results are provided within the manuscript.

Authors' Contributions
The author(s) were responsible for the conception and design of the study, the development of definitions, the establishment and proof of theoretical results, and the preparation of the manuscript. All authors reviewed and approved the final version of the paper.

REFERENCES

[1]Andrijevic. D, (1996), Semi-pre-open sets, Matematicki Versnik, 38, 24-32.
[2]Balachandran. K, Sundarm. P and Maki. H (1991), On genralized continuous maps in topological spaces, Memoirs of the faculty of science kochi university series mathematics, 12, 5-13.
[3] Bhatacharya. P and Lahari. B. K, (1987), semi generalized closed sets in topology, Indian Journal of Mathematics 29 (3), 375-382.
[4]Cagman. N., Karatas. S., Enginoglu. S., Aydin. T, On soft Topology, El-Cezeri Jounal of Science and Engineering, vol: 2 No: 3, pp: 23-38 (2015).
[5] Csazar. A, Generalized topology, generalized continuity, Acta. Math. Hungar. 96(2002), 351-357.
[6]Crossley. S. G. Hildebrand (1972), semi topological properties, Fundamenta Mathematicae, 20, 472-482.
[7] Gozde Yaylali, Bekir Tonay, Results on Soft continuous functions in soft topological spaces quipped with soft scott topology, Journal of non-linear sciences and applications, 10(2017), 1183-1194.
[8] Gomathi. N, Meenakumari. N, Indira. T On g**-closed sets in topological spaces, IJMA, Vol: 6, Issue: 1 Pp:85-90.
[9] Gomathi. N and Indira. T, On Soft generalized** Continuous functions in soft topological spaces, Mathematical Statistician and Engineering applications.,Vol: 71, No: 4 (2022).
[10] James.R. Munkres(1990), Topology, Prentice hall of India private limited, New Delhi.
[11] Kharal. A, Ahmed. B, Mappings of Soft classes, New.Math.Nat.,Comput.,7(3), 2011, Vol: 37, No:1, 17-21.
[12] Levine. N, On generalized closed sets in topology, Rend.Circ.Math.Palerno, 19(2) (1970) 89-96.,
[13] Levine. N, Semi open sets and semi continuity in topological spaces, Amer.Math Monthly,70(1963).36-41.
[14] Maji. P. K and Roy. A. R , An applications of soft sets in a decision making problem, Computers and mathematics with applications 44(2002), 1077-1083.
[15] Maki.H , Devi.R and Balachandran.K(1994), Associated topologies of generalized α -closed sets and α -generalized closed sets, Memoirs of faculty of science kochi university series, A Mathematics 15, 51-63.
[16] Mareay. R and Medhat. T and Manal. E. Ali, Soft sets and soft topological spaces via its applications, Kafrelsheikh Journal of information sciences, Vol: 3, Issue:1, 2022, PP.1-7.
[17] Mohamed Shabir and Munazza Naz., On soft topological spaces, Computers and Mathematics with Applicatiions 61(2011) 1786-1799.
[18] Molodtsov. D., Soft Set Theory First Results, Computers and Mathematics with Applications, vol:37, pp: 19-31 (1999).
[19] Pawlak.Z(2002), Rough set Theory and its applications, Journal of Telecommunicators and Information Technology, 3, 7-10.
[20] Peyghan. E, Somadi. B & Tayebi. A, Some results related to soft topological spaces, Facta Universitatis (Nis), Vol:29, No: 4 (2014), 325-336.
[21] Sabir Hussain., Bashir Ahmad., Some Properties of Soft Topological Spaces, Computers and Mathematics with Applications, vol:62, pp. 4058-4067 (2011).

[22] Sharma. J. N and Chauhan. J. P (2017), Topology (General and Algebraic), Krishna Prakshan media private limited, Uttar Pradesh.
[23] Won Keun Min., A note on soft Topological spaces, Computers and Mathematics with Applications, Vol:62, pp.3254-3528 (2011).
[24] Zadeh. L. A, Fuzzy sets, Information and Control 8 (1965), 338-353, DOI: 10.1016/S0019-9958(65)90241-X.
[25] Zorlutna. I, M. Akdag, W. K. Min, S. Atmaca, Remarks on soft topological spaces, Annals of Fuzzy Mathematics and Informatics. Vol:3, No:2, (April 2012), PP. 171-185.